Errata to TR96-026

The purpose of this comment is to correct the numerical error in the lower bound for the clique function.

The correction concerns the claim (at the end of the proof of Theorem 3.1 on page 6) that
\[ \#_s(X^0) \leq (k - 2)^{m-s}. \]
Actually, \( \#_s(X^0) \) is the number of \((k - 1)\)-colorings with at least \( s/2 \) pre-determined values on some fixed set of \( s \) vertices, and hence, we can ensure only that \( \#_s(X^0) \leq (k - 1)^{m-s/2} \).

This correction leads to worse lower bound for CLIQUE_{m,k} : arguing exactly as in the proof of Theorem 3.1, we get the lower bound \( \min \left\{ \left( \frac{m}{2}, \frac{m}{(m/2)} \right) \left( \frac{m}{(m/2)} \right) \right\} \), which, for \( s \coloneqq \lceil m/2k \rceil \) and \( r \coloneqq \lfloor \sqrt{n}/2 \rfloor \), is \( 2^{\Omega((m/2k, \sqrt{n}))} \). Thus, Theorem 3.1 should sound as follows:

**Theorem 3.1.** Let \( \ell \leq \min \left\{ \frac{m}{2}, \frac{m}{\sqrt{n}} \right\} \), and let \( C \) be a circuit with unbounded fan-in AND, OR gates and arbitrary monotone Boolean functions of fan-in \( \ell \) at the bottom. If \( C \) computes CLIQUE_{m,k} then \( C \) has size \( 2^{\Omega(\min(m/k, \sqrt{n}))} \). In particular, if \( k \approx m^{2/3} \) then the size of \( C \) is \( 2^{\Omega(n^{1/6})} \).

Corresponding small changes should be made in the abstract and Introduction; \( \Omega(n^{1/4}) \) must be replaced by \( \Omega(n^{1/6}) \), and the bound \( 2^{\Omega(k)} \) by \( 2^{\Omega(k^{1/2})} \).

Thus, our criterion (Theorem 2.1) gives actually the same lower bound \( 2^{\Omega(k^{1/2})} \) for CLIQUE_{m,k} as that proved by Alon and Boppana. The only difference is that our bound holds for slightly larger values of \( k \), namely - for any \( k \leq m^{2/3} \) (rather than \( k \leq (m/8 \log m)^{3/3} \)).

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