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The Complexity of Unary Knapsack with Signed Repetition

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Theorem 1 *The problem Unary Knapsack with Signed Repetition (UKSR) is in AC^0.*

Proof. Note that UKSR is equivalent to the question whether a linear Diophantine equation in several variables has a solution in integers. A necessary and sufficient condition for a positive answer is that the greatest common divisor of all coefficients ($y_1, \ldots, y_n$ in UKSR) divides the constant (y in UKSR) [1, pp. 94–98].

Let the input be $0^y, 0^{y_1}, \ldots, 0^{y_n}$, where each 0-block is followed by a marker. We outline the construction of a constant-depth, polynomial size circuit deciding the property described above. Let the input length be m. For every input segment from position i to j, $2 \leq i < j \leq m$ and every $2 \leq q \leq m$ design a sub-circuit $c_{i,j,q}$ that returns true if and only if the segment does not encode one of the y_i (there is no marker at position i or j or some symbol at positions $i+1 \ldots j-1$ does not equal 0) or q divides $j-i-1$. For every q form the and of all $c_{i,j,q}$ to obtain d_q. In a similar way construct circuits d'_q for the divisors of y. Form an and over all $\neg d_q \vee d'_q$ to obtain the output. □

References

[1] Ivan Niven and Herbert S. Zuckerman. *An Introduction to the Theory of Numbers*, J. Wiley & Sons, New York - London, 1960.

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