The Complexity of Unary Knapsack with Signed Repetition

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Theorem 1 The problem Unary Knapsack with Signed Repetition (UKSR) is in AC^0.

Proof. Note that UKSR is equivalent to the question whether a linear Diophantine equation in several variables has a solution in integers. A necessary and sufficient condition for a positive answer is that the greatest common divisor of all coefficients \((y_1, \ldots, y_n)\) in UKSR divides the constant \((y)\ in UKSR) \([1, pp. 94-98]\).

Let the input be \(0^p, 0^q_1, \ldots, 0^q_r\), where each 0-block is followed by a marker. We outline the construction of a constant-depth, polynomial size circuit deciding the property described above. Let the input length be \(m\). For every input segment from position \(i\) to \(j\), \(2 \leq i \leq j \leq m\) and every \(2 \leq q \leq m\) design a sub-circuit \(c_{i,j,q}\) that returns true if and only if the segment does not encode one of the \(y_i\). (There is no marker at position \(i\) or \(j\) or some symbol at positions \(i+1 \ldots j-1\) does not equal 0) or \(q\ divides j-i-1\). For every \(q\) form the and of all \(c_{i,j,q}\) to obtain \(d_q\). In a similar way construct circuits \(d_q^p\) for the divisors of \(y\). Form an and over all \(\neg d_q \lor d_q^p\) to obtain the output.

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References


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