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A note on FewP $\subseteq$ EP

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We give a simple proof of Borchert, Hemaspaandra, and Rothe's result that FewP $\subseteq$ EP. For definitions and a list of important problems in EP the reader is referred to Borchert et al [1].

Proof. Let $L \in$ FewP via an NTM M that runs in time $p(n)$ and has no more than $q(n)$ accepting paths on any input of length n. Let M' behave as follows on input x: guess an odd number k between 1 and $q(|x|)$; guess a set S consisting of k strings of length $p(|x|)$ or less; accept if every string in S is an accepting computation of M on input x. If M has 0 accepting paths on input x, then clearly M' has 0 accepting paths on input x. If M has m accepting paths on input x, where $m > 0$, then M' has $\sum_{1\leq k\leq m, k\text{odd}} \binom{m}{k} = 2^{m-1}$ accepting paths on input x. Thus M' witnesses that $L \in$ EP. ∎

Note 1. Observe the similarity between our proof that FewP $\subseteq$ EP and Schöning's proof [2] that FewP $\subseteq \oplus$P, in which M' guesses any nonempty set of accepting paths.

Note 2. Our technique can be seen as a simple extension of Borchert et al's general technique. Whereas they apply a positive weight to each nonempty set of accepting paths, we apply a positive weight to singleton sets of accepting paths and *nonnegative* weights to larger sets of accepting paths.

Note 3. Although Borchert et al's general method clearly allows us to convert FewP machines to NP machines that accept with a number of accepting paths that is a power of b for any fixed integer $b \geq 2$, it is interesting to obtain a closed form. Let M' behave as follows on input x: guess an integer k between 1 and $q(|x|)$; guess an integer between 1 and $\frac{1}{b}\left((b-1)^k - (-1)^k\right)$; guess a set S consisting of k strings of length $p(|x|)$ or less; accept if every string in S is an accepting computation of M on input x. If M has m accepting paths on input x, where $m > 0$, then M' has $\sum_{1\leq k\leq m} \frac{1}{b}\left((b-1)^k - (-1)^k\right)\binom{m}{k} = b^{m-1}$ accepting paths on input x.

References

[1] B. Borchert, L. A. Hemaspaandra, and J. Rothe. Powers-of-two acceptance suffices for equivalence and bounded ambiguity problems. TR 96-045, Electronic Colloquium on Computational Complexity, http://www.eccc.uni-trier.de/eccc/, 1996.

[2] U. Schöning. The power of counting. In *Proceedings of the 3rd Annual Conference on Structure in Complexity Theory*, pages 2–18. IEEE Computer Society Press, June 1988.

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