A note on $\text{FewP} \subseteq \text{EP}$

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October 17, 1996

We give a simple proof of Borchert, Hemaspaandra, and Rothe’s result that $\text{FewP} \subseteq \text{EP}$. For definitions and a list of important problems in EP the reader is referred to Borchert et al [1].

**Proof.** Let $L \in \text{FewP}$ via an NTM $M$ that runs in time $p(n)$ and has no more than $q(n)$ accepting paths on any input of length $n$. Let $M'$ behave as follows on input $x$: guess an odd number $k$ between 1 and $q(|x|)$; guess a set $S$ consisting of $k$ strings of length $p(|x|)$ or less; accept if every string in $S$ is an accepting computation of $M$ on input $x$. If $M$ has 0 accepting paths on input $x$, then clearly $M'$ has 0 accepting paths on input $x$. If $M$ has $m$ accepting paths on input $x$, where $m > 0$, then $M'$ has $\sum_{1 \leq k \leq m, k \text{odd}} \binom{m}{k} = 2^{m-1}$ accepting paths on input $x$. Thus $M'$ witnesses that $L \in \text{EP}$.

**Note 1.** Observe the similarity between our proof that $\text{FewP} \subseteq \text{EP}$ and Schöning’s proof [2] that $\text{FewP} \subseteq \text{⊕P}$, in which $M'$ guesses any nonempty set of accepting paths.

**Note 2.** Our technique can be seen as a simple extension of Borchert et al’s general technique. Whereas they apply a positive weight to each nonempty set of accepting paths, we apply a positive weight to singleton sets of accepting paths and *nonnegative* weights to larger sets of accepting paths.

**Note 3.** Although Borchert et al’s general method clearly allows us to convert $\text{FewP}$ machines to NP machines that accept with a number of accepting paths that is a power of $b$ for any fixed integer $b \geq 2$, it is interesting to obtain a closed form. Let $M'$ behave as follows on input $x$: guess an integer $k$ between 1 and $q(|x|)$; guess an integer between 1 and $\frac{1}{b} \left( (b-1)^k - (-1)^k \right)$; guess a set $S$ consisting of $k$ strings of length $p(|x|)$ or less; accept if every string in $S$ is an accepting computation of $M$ on input $x$. If $M$ has $m$ accepting paths on input $x$, where $m > 0$, then $M'$ has $\sum_{1 \leq k \leq m} \frac{1}{b} \left( (b-1)^k - (-1)^k \right) \binom{m}{k} = b^{m-1}$ accepting paths on input $x$.

**References**


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