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Irregular Assignments of the Forest of Paths

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Abstract

Let G be a graph on n vertices. An *irregular assignment* of G is a labelling $f : E \to \{1, 2, ..., m\}$ of the edge-set of G such that all of the induced vertex labels computed as $f(v) = \sum_{v \in e} f(e)$ are distinct. The minimal number m for which this is possible is called the *minimal irregularity strength* $s_m(G)$ of G. The case where all paths are of length 2 is considered by Aigner and Triesch by using decompositions of the additive group $\mathbf{Z}_r$. In this paper we have investigated irregular assignments of the forests of paths of regular and irregular lengths.

1 Introduction

Appearantly irregular assignment of a graph was initiated by Behzad and Chartrand [1]. The problem is to draw a graph with no two points have the same degree. Clearly the only totally irregular graph is the trivial graph K_1. If this definition is applied to the multi-graphs without self-loops (totally irregular [2]) or to vertex degrees of the neighborhood of a vertex of a graph (highly irregular [3],[4],[5]) then the problem of finding an assignment (edge-labelling) which gives number of parallel edges between the pair of vertices, becomes non-trivial and challengeable. The closely related formulation of the problem is given by Chartrand *et. al* which is the following [2]:
Let G be a graph on n vertices. An irregular assignment of G is a labelling $f : E(G) \to \{1, 2, ..., m\}$ of the edge-set of G such that when the induced vertex label computed by $f(v) = \sum_{\forall u, (v,u) \in E(G)} f(v, u) \quad \forall v \in V(G)$, all vertex labels are distinct. The minimal number m for which this is possible is called the *irregularity strength* $s_m(G)$ of G.

The simplicity of the statement (not the solutions) of this problem attracts many people to find irregular assignments of the many classes of graphs; see for example the results, open problems and conjectures in the excellent survey by J.Lehel [2].

In this paper we will give further results on the irregular assignment of the forest of paths of regular (same) and irregular (different) lengths. Let us denote by $F(n; k)$ k disoint paths of length n. While the former generalize the result of Aigner and Triesch [5], the later gives algorithmic aspects of the problem which has some similarity with the one-dimensional bin-packing problem.

2 Forest of Regular Paths

Let us first give several simple bounds on the strength of the forest of paths:

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Theorem 1a. [5]: *Let F be a forest on $n \geq 3$ vertices without isolated edges and at most one isolated vertex. Then $s(F) \leq n+1$.*

Theorem 1b. *Let $F(P_{i_1}, P_{i_2}, ..., P_{i_k})$ be the forest of paths $P_{i_1}, P_{i_2}, ..., P_{i_k}$. Then the lower bound for any irregular assignment of F is*

$$max\{\frac{|V|}{2}, |V_e|\} \leq s_m(F(P_{i_1}, P_{i_2}, ..., P_{i_k}))$$

where $|V_e|$ is the number of the end-vertices in F.

Proof. In case $|V_e| \geq \frac{|V|}{2}$, e.g., $k \leq 3$, the edge labels $1, 2, ..., |V_e|$ will have to be assigned to the end-edges of F in any irregular assignments f. Assume $\frac{|V|}{2} > |V_e|$ and $s(F) < \frac{|V|}{2}$ and consider the induced vertex labels F_v under f: $F_v = \{1, 2, ..., |V_e|, ..., X\}$. $\max f(v) \leq |V|$ if $s(F) = \frac{|V|}{2}$ since $max\{degree(v)\} = 2$. Therefore the induced vertex labels set $F_v = \{1, 2, ..., |V|\}$ is obtained only if $s(F) = \frac{|V|}{2}$. Hence the proof of the theorem follows. □

Theorem 2[8]. *Let P_n denotes the path of length n then the strength is given by*

$$s(P_n) = \begin{cases} \lceil \frac{n}{2} \rceil & \text{if } n \equiv 3 \ (mod 4) \\ \lceil \frac{n}{2} \rceil + 1 & \text{if } n \equiv 0, 1, 2 \ (mod 4), n \neq 1. \end{cases}$$

Kinch and Lehel [6] have shown that the irregularity number of $F(2; k)$ is

$$\lceil \frac{15k-1}{7} \rceil \leq s(F(2; k)) \leq \lceil \frac{15k-1}{7} \rceil + 1$$

and Kinch conjectured that actually $s(F(2; k)) = \lceil \frac{15k-1}{7} \rceil$ for all $k > 0$. M. Aigner and E. Triesch [5] have shown latter the following:

Theorem 3,[5]. *The irregularity strength of disjoint k paths of length 2 is*

$$s(F(2; k)) = \lceil \frac{15k-1}{7} \rceil$$

unless when $k \equiv 15, 22$ (mod 28) in which case $s(F(2; k)) = \lceil \frac{15k-1}{7} \rceil + 1$.

Theorem 4. *If $n \equiv 3$ (mod 4), i.e.,$n = 3 + 4p, p \geq 1$, then the strength of the forest $F(n; k)$ of k paths of lengths n is*

$$s_m(F(n; k)) = \frac{k(n+1)}{2}$$

.

Proof. We will prove the theorem by construction. Let $n = 4p + 3$ and $\Delta = 2k$. Label the edges of the ith path P_i from the left-side with integers

$$i, \Delta, i + \Delta, 2\Delta, i + 2\Delta, 3\Delta, i + 3\Delta, ..., (p + 1)\Delta$$

and from the right-side of P_i with integers

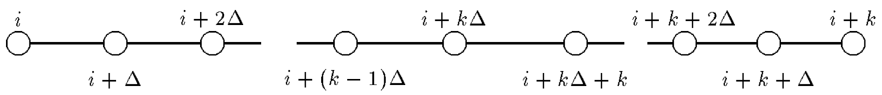
$$i + k, \Delta, i + k + \Delta, 2\Delta, i + k + 2\Delta, 3\Delta, i + k + 3\Delta, ..., i + k + p\Delta.$$

Clearly the last labels i.e., $(p + 1)\Delta$ and $i + k + p\Delta$ of the above sequences incident on the edges of the path P_i. Then the induced vertex labels of $P_i, i = 1, 2, ..., k$ which are the sum of the edge labels at the vertices are found to be

$$\{i, i + \Delta, i + 2\Delta, i + 3\Delta, i + 4\Delta, ..., i + k + p + (p + 1)\Delta, i + k + 2p\Delta, ..., i + k + 3\Delta, i + k + 2\Delta, i + k + \Delta, i + k\}, i = 1, 2, ..., k$$

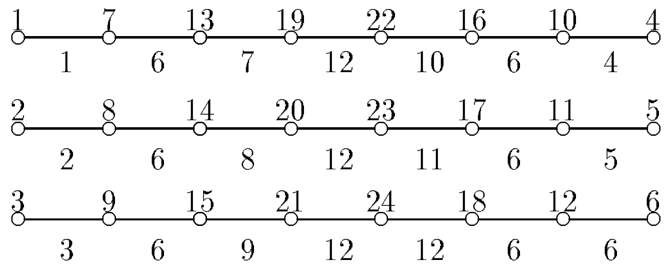
If we re-arrange the integers in the set in increasing order we get $\{1, 2, ..., (n + 1)k\}$ which shows that the edge assignment (labelling) f is irregular with strength $s(F(n; k)) = \frac{k(n+1)}{2}$. □

Our irregular edge-assignment is illustrated for $n = 7, k = 3$.



$i = 1, 2, ..., k,\ \ \Delta = 2k$ and $n \equiv 3 \pmod 4$

(a)



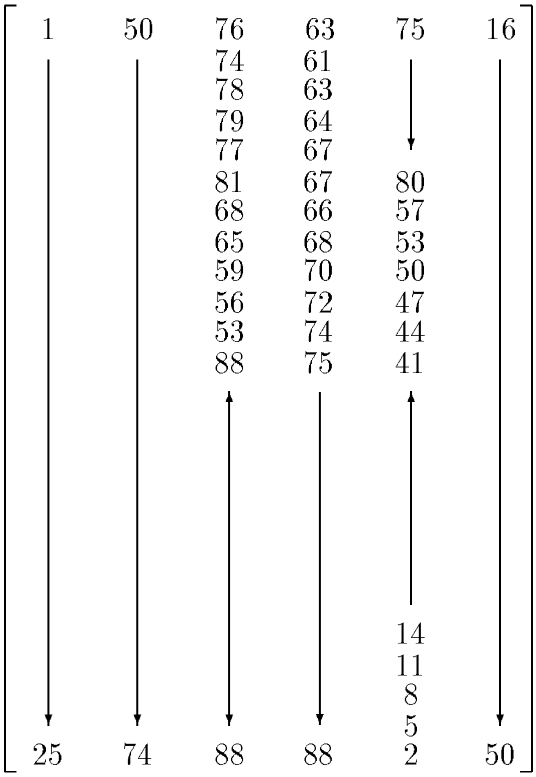
(b) $s(F(7; 3)) = 12$

Figure 1

In the light of the Theorem 3, (cf., Aigner and Triesch [5]) the difficult case in the genearlization of the forest of paths of length n would be the case when $n \equiv 2 \pmod 4$. Next we assert a claim which gives a very sharp upper bound (in fact conjectured exact) values on the strengths of $F(n; k)$ when $n \equiv 2 \pmod 4$.

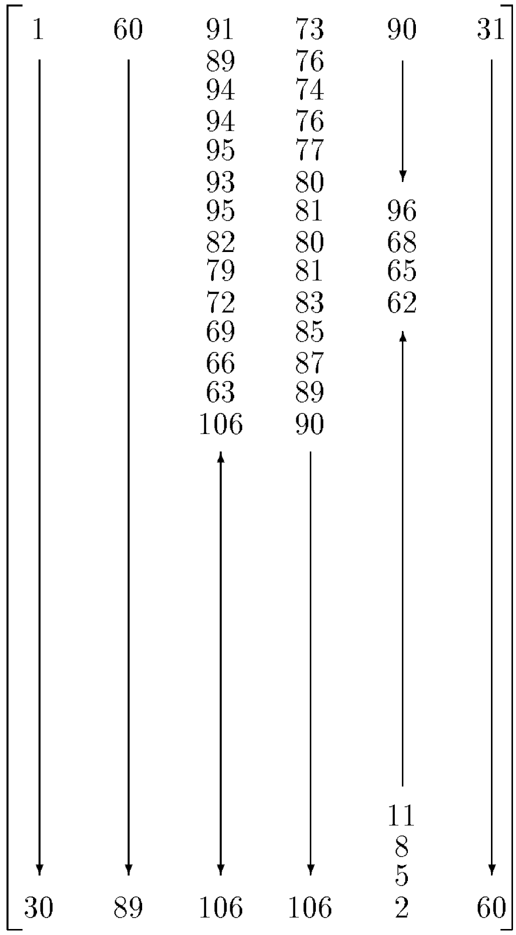
Claim 5. *If $n \equiv 2 \pmod 4$, $n \neq 2$ then the attainable upper bound for the strength of the forest $F(n;k)$ is $s(F(n;k)) = \lfloor \frac{(n+1)k}{2} \rfloor + 1$.*

We have given matrices $E_{k\text{x}n}$ associated with the irregular assignment of $F(n;k)$ for $n = 6$ and $k = 1, 2, \ldots, 30$ that verify the statement of the claim above but we could not able to generalize it to any $n \neq 2$ and k. The construction is illustrated for $n = 6, k = 25$ and $n = 6, k = 30$ by the matrices shown below:



$E_{25\text{x}6}$

$s(F(6;25)) = 88$, missing integer= 145



$E_{30\text{x}6}$

$s(F(6;30)) = 106$, missing integers= $174, 178$

Theorem 6. *If $n \equiv 1 \pmod 4$ and $k \equiv 0 \pmod 2$, then the strength of the forest $F(n;k)$ of k paths of lengths n is*

$$s_m(F(n;k)) = \frac{k(n+1)}{2}$$

Proof: Proof by construction of a suitable matrix which gives irregular edge assignments of $F(n;k)$. The following matrix gives the irregular edge-labellings which results in distinct induced vertex-labels, where rows of the matrix correspond to labelled paths in the forest and the "arrows" in the matrix denotes the consecutive integers.

$$\left[\begin{array}{cccccccccccc}
1 & 2k & 2k+1 & 4k & 4k+1 & \lfloor\frac{n}{2}\rfloor k & \lfloor\frac{n}{2}\rfloor k+1 & \lfloor\frac{n}{2}\rfloor k+1 & 4k+1 & 2k+1 & k+1 & k+1 \\
\Big\downarrow & \Big\downarrow & \Big\downarrow & \Big\downarrow & \Big\downarrow \cdots & \Big\downarrow & \Big\downarrow & \Big\downarrow \cdots & \Big\downarrow & \Big\downarrow & \downarrow \frac{3k}{2} \; \frac{5k}{2} \downarrow & \Big\downarrow \\
k & 3k-1 & 3k & 5k-1 & 5k & \lceil\frac{n}{2}\rceil k-1 & \lceil\frac{n}{2}\rceil k & \lceil\frac{n}{2}\rceil k & 5k & 3k & 3k & 2k
\end{array}\right]$$

□

Theorem 7. *If $n \equiv 1$ (mod 4) and $k \equiv 1$ (mod 2) then*

$$s(F(n;k)) \leq \frac{k(n+1)}{2} + 1.$$

Proof: Let $n \equiv 1 \pmod 4$ and $k \equiv 1 \pmod 2$. Consider the matrix E_{kxn} corresponding to the edge labels of the paths in $F(n;k)$:

$$\left[\begin{array}{ccccccccccccc}
1 & 2k+2 & 3k+1 & 4k+2 & 5k+1 & \frac{(n-1)k}{2}+1 & \frac{(n+1)k}{2}+1 & \frac{(n-3)k}{2}+1 & 5k+1 & 3k+2 & 3k+1 & k+1 & k+1 \\
\Big\downarrow & \Big\downarrow & \Big\updownarrow & \Big\downarrow & \Big\updownarrow \cdots & \Big\downarrow & \Big\updownarrow & \Big\downarrow \cdots & \Big\updownarrow & \Big\downarrow & \Big\updownarrow & \Big\downarrow & \Big\downarrow \\
k & 3k+1 & 3k+1 & 5k+1 & 5k+1 & \frac{(n+1)k}{2}+1 & \frac{(n+1)k}{2}+1 & \frac{(n-1)k}{2}+1 & 5k+1 & 4k+1 & 3k+1 & 2k & 2k
\end{array}\right]$$

where for $x < y$, the symbol "$x \longrightarrow y$" indicates that all integers from x to y and the symbol "$x \longleftrightarrow x$" indicates that all integers in the range of the double-end arrow are equal to x. In E_{kxn} entries of i'th row corresponds to edge labels of the path $p_i, 1 \leq i \leq k$. Now the induced vertex labels can be found by taking pairwise sums of adjacent and the first and last entries of each row. It can be verified that except the integers $2k+1$ and $5k+2$ all vertex labels in the range $[1, (n+1)k+2]$ are presented. Therefore we have $s(F(n;k)) \leq \frac{(n+1)k}{2} + 1$. In fact we claim (unproved) that $s(F(n;k)) = \frac{(n+1)k}{2} + 1$. □.

Theorem 8. *If $n \equiv 0$ (mod 4) then the strength of the forest $F(n;k)$ of k paths of length n is given by*

$$s(F(n;k)) = \lfloor \frac{k(n+1)}{2} \rfloor + 1$$

where $\lfloor x \rfloor$ *is the largest integer smaller than* x*.*

Proof: We will give the proof by construction of suitable matrices $\mathrm{E}_{k\mathrm{x}n}$ associated with the irregular edge-assignment of the paths in $F(n;k)$. Let

$$\mathrm{E}_{k\mathrm{x}n} = [f(e_{i,j})]$$

where the entry $f(e_{i,j})$ denotes the label of the j'th edge (when counting the edges from the left) in the i'th path p_i. For example consider the matrix $\mathrm{E}_{6\mathrm{x}4}$

$$\mathbf{E}_{6\mathrm{x}4} = \begin{bmatrix} 1 & 13 & 10 & 7 \\ 2 & 13 & 11 & 8 \\ 3 & 13 & 16 & 9 \\ 4 & 14 & 16 & 10 \\ 5 & 15 & 16 & 11 \\ 6 & 16 & 16 & 12 \end{bmatrix}$$

It follows from the matrix $\mathrm{E}_{6\mathrm{x}4}$ that all the induced vertex-labels with exception 13 and 21 from 1 to 32 are presented. In the matrices the "arrow" on the columns denotes that integers along the direction of the arrow are increasing one-by-one and the "box" on the columns denotes that integers within the box are all same. In general we distinguish two cases.

Case a: $n \equiv 0 \pmod 4$, $k \equiv 0 \pmod 2$.

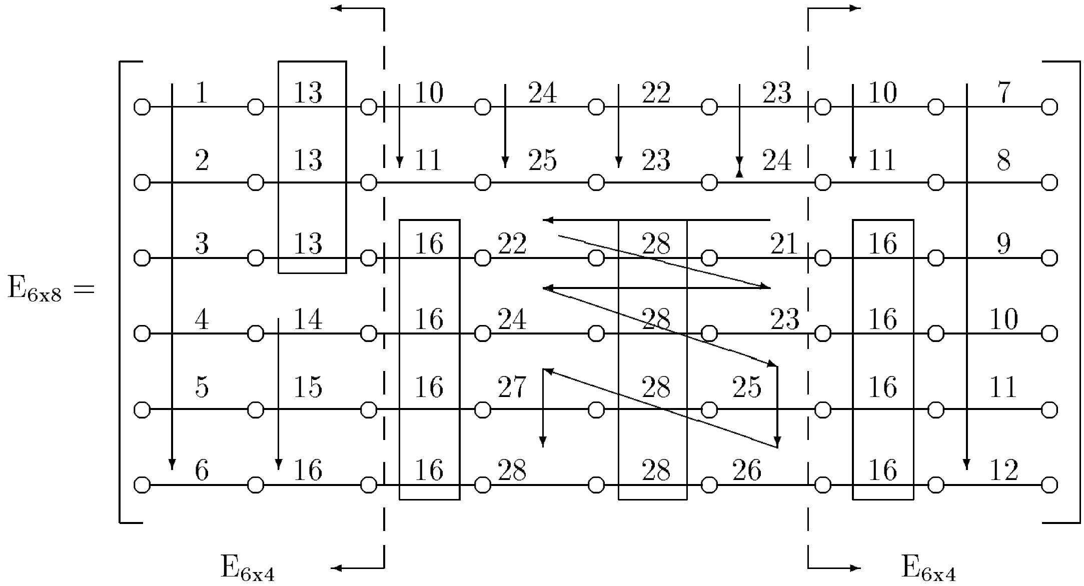
Let $n = 4, k \equiv 0 \pmod 2$ and consider the following matrix:

$$
E_{k\mathrm{x}4} = \begin{bmatrix}
1 & 2k+1 & \frac{3k}{2}+1 & k+1 \\
2 & 2k+1 & \frac{3k}{2}+2 & k+2 \\
\vdots & \vdots & \vdots & \vdots \\
\frac{k}{2}-1 & 2k+1 & 2k-1 & \frac{3k}{2}-1 \\
\frac{k}{2} & 2k+1 & \frac{5k}{2}+1 & \frac{3k}{2} \\
\frac{k}{2}+1 & 2k+2 & \frac{5k}{2}+1 & \frac{3k}{2}+1 \\
\frac{k}{2}+2 & 2k+3 & \frac{5k}{2}+1 & \frac{3k}{2}+2 \\
\vdots & \vdots & \vdots & \vdots \\
k & \frac{5k}{2}+1 & \frac{5k}{2}+1 & 2k
\end{bmatrix}
$$

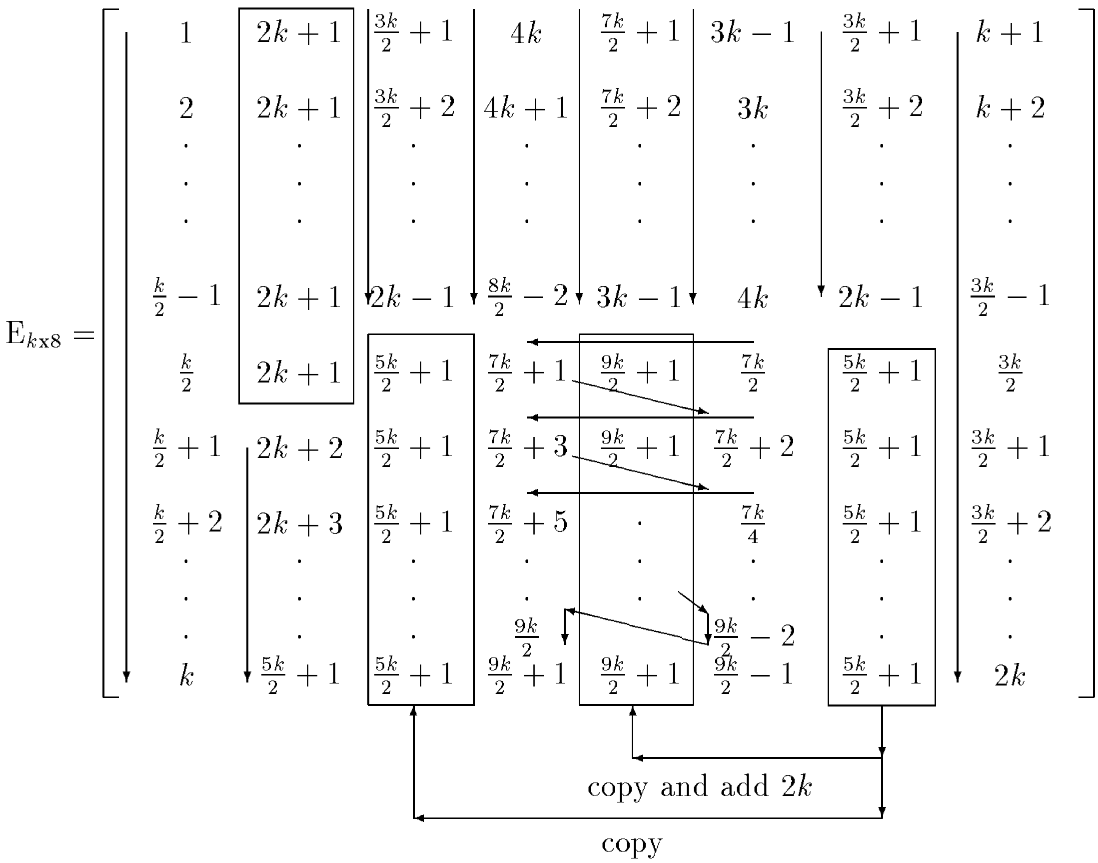
From $E_{k\mathrm{x}4}$ we see that the missing vertex labels in the induced vertex labels range $[1, 5k+2]$ are $2k+1$ and $\frac{7k}{2}$. Hence the edge labelling of $F(4;k)$ when $k \equiv 0 \pmod 2$ given by $E_{k\mathrm{x}4}$ corresponds to an irregular assignment with strength $s(F(4;k)) \leq \frac{5k}{2}+1$. On the otherhand by Theorem 1 we have

$$max\{\frac{|V|}{2}, |V_e|\} = \frac{5k}{2} \leq s(F(P_1, P_2, ..., P_k)).$$

Hence the difference between these lower and upper bounds is only 1. Therefore we can safely claim that $s(F(4;k)) = \frac{5k}{2}+1$. Next we will show construction of the matrices $E_{k\mathrm{x}n}, n \equiv 0 \pmod 4 > 4, k \equiv 0 \pmod 2$ from $E_{4\mathrm{x}k}$. The method of the construction can be called *tear-modify-glue* and best examplify on our running example i.e., $E_{6\mathrm{x}4}$:



We then easily generalize this construction to the matrix E_{kx8} as shown below:



Again from the matrix E_{kx8} we see that the missing vertex labels in the range $[1, 9k+2]$ of vertex labels are $2k+1$ and $\frac{7k}{2}$. We can easily generalize the construction given above for higher values of $n \equiv 0 \pmod 4$, say for $n = 4(p-1)$, by partioning (tearing) the matrix $E_{kx4(p-1)}$ into two *left* and *right* submatrices as

$$\mathrm{E}_{k\mathrm{x}4(p-1)} = [\mathrm{E}^{l}_{k\mathrm{x}2(p-1)}|\mathrm{E}^{r}_{k\mathrm{x}2(p-1)}]$$

and then inserting the new submatrix $\hat{\mathrm{E}}_{k\mathrm{x}4}$ (see for example the matrix $\mathrm{E}_{k\mathrm{x}8}$) in between $\mathrm{E}^{l}_{k\mathrm{x}2(p-1)}$ and $\mathrm{E}^{r}_{k\mathrm{x}2(p-1)}$:

$$\mathrm{E}_{k\mathrm{x}4(p-1)} = [\mathrm{E}^{l}_{k\mathrm{x}2(p-1)}|\hat{\mathrm{E}}_{k\mathrm{x}4}|\mathrm{E}^{r}_{k\mathrm{x}2(p-1)}]$$

The modified submatrix $[\hat{\mathrm{E}}_{k\mathrm{x}4}]^T$ is shown for $n = 4p$ below:

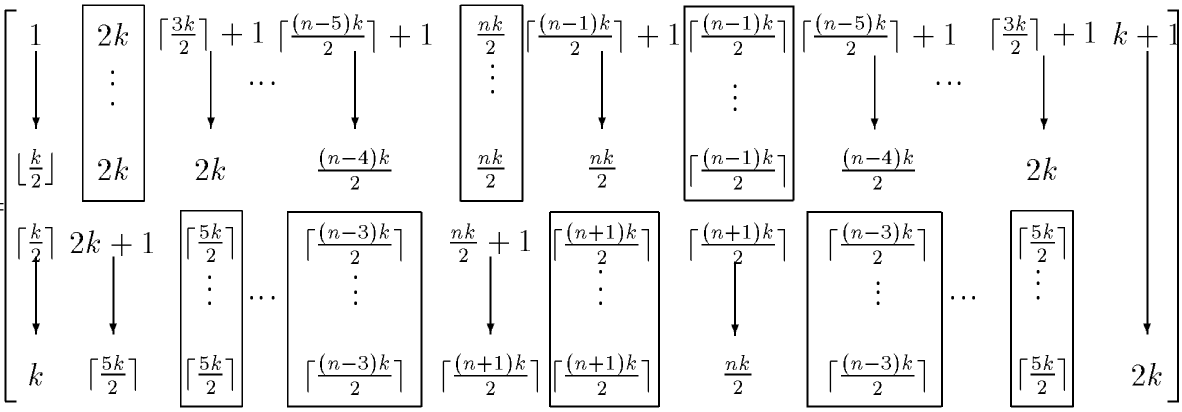
$$\hat{\mathrm{E}}_{k\mathrm{x}4} = \begin{bmatrix}
\frac{(n-5)k}{2}+1 & \frac{nk}{2} & \frac{(n-1)k}{2}+1 & \frac{nk}{2}-1 \\
\frac{(n-5)k}{2}+2 & \frac{nk}{2}+1 & \frac{(n-1)k}{2}+2 & \frac{nk}{2} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{(n-4)k}{2}-1 & \frac{(n+1)k}{2}-2 & \frac{nk}{2}-1 & \frac{(n+1)k}{2}-3 \\
\frac{(n-3)k}{2}+1 & \frac{(n-1)k}{2}+1 & \frac{(n+1)k}{2}+1 & \frac{(n-1)k}{2} \\
\frac{(n-3)k}{2}+1 & \frac{(n-1)k}{2}+3 & \frac{(n+1)k}{2}+1 & \frac{(n-1)k}{2}+2 \\
\vdots & \vdots & \vdots & \vdots \\
\frac{(n-3)k}{2}+1 & \frac{(n+1)k}{2} & \frac{(n+1)k}{2}+1 & \frac{(n+1)k}{2}-2 \\
\frac{(n-3)k}{2}+1 & \frac{(n+1)k}{2}+1 & \frac{(n+1)k}{2}+1 & \frac{(n+1)k}{2}-1
\end{bmatrix}$$

Hence we obtain an irregular assignment f of $F(4p; k)$ with strength

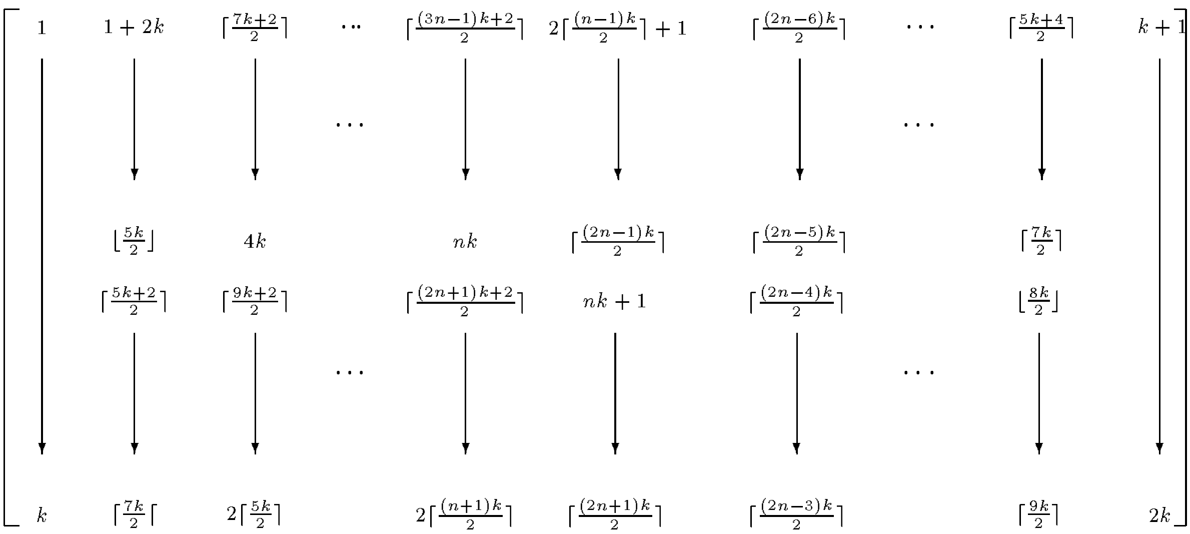
$$s(F(4p; k)) = \frac{k(4p+1)}{2} + 1.$$

The induced vertex-labels go from 1 to $(4p+1)k+2$ with exceptions of integers $2k+1$ and $\frac{7k}{2}$.
Case b: $n \equiv 0 \pmod 4$, $k \equiv 1 \pmod 2$.
This case is very similar to that of *Case a*. We will only give the structure of the matrix $\mathrm{E}_{k\mathrm{x}n}$ corresponding to the irregular assignment of $F(n; k)$:

$\mathrm{E}_{k\mathrm{x}n} =$



In order to show that the above matrix corresponds to the irregular edge assignments of $F(n;k)$ we compute the induced-vertex labels as the entries of another matrix $f[\mathrm{V}]_{k\mathrm{x}(n+1)}$ which is obtained from the pair-wise sums of the entries of each row of the matrix $\mathrm{E}_{k\mathrm{x}n}$. The matrix $[\mathrm{V}]_{k\mathrm{x}(n+1)}$ is shown below:

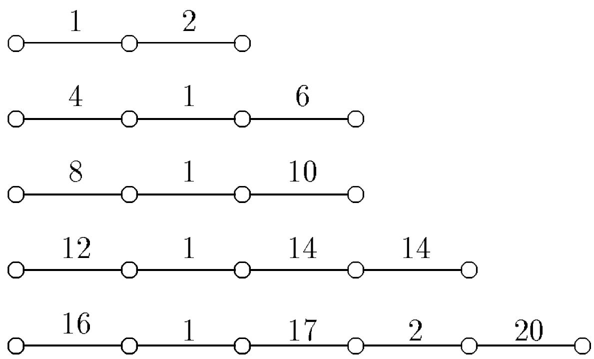


In the matrix $[\mathrm{V}]_{k\mathrm{x}(n+1)}$ the induced vertex labels go from 1 to $2\lceil\frac{(n+1)k}{2}\rceil$ except the integer $\lfloor\frac{k}{2}\rfloor + 2k + 1$. Hence the strength of the forest $F(n;k)$ for $n \equiv 0 \pmod 4$ and k odd, is $\lfloor\frac{(n+1)k}{2}\rfloor + 1$. $\square$

3 Forest of Irregular Paths

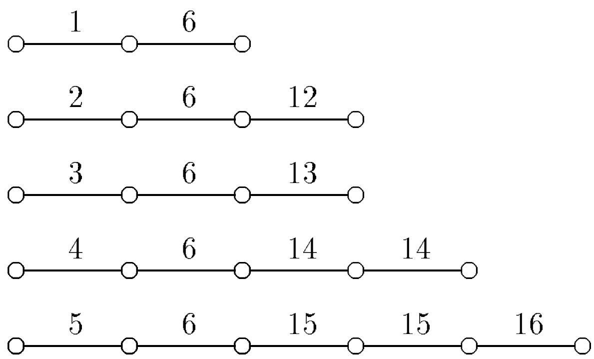
In this section we will consider the problem of finding irregular edge-assignments for the set of paths of different lengths, where *different* means that at least two paths in the forest have different in lengths. As it will be shown while if the distribution of the paths has some property i.e., the lengths of the paths are given in linear increasing order then exact value of the strength can be found. However the problem of finding the exact value for the strength in general is an algorithmic problem and we can only give upper bounds for the two methods. The first method is "*serial-first-fit*" (SFF) in which the edge-assignment of each paths is considered one-by-one and the second method is "*parallel-first-fit*" (PFF) in which one edge of each path is labelled at the same time in the forest of paths. Let us denote the forest of k paths of lengths $l_1, l_2, ..., l_k$ by $F(l_1, l_2, ..., l_k)$. Before we go further consider $F(2, 3, 3, 4, 5)$.

Example (a). *Irregular edge-assignment by SFF*



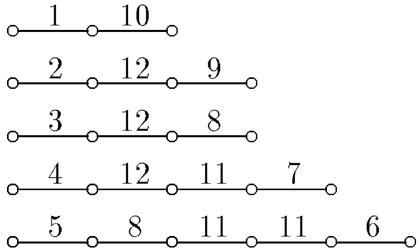
As it can be seen from the above labelling $s(F(2, 3, 3, 4, 5)) \leq 20$.

Example (b). *Irregular edge-assignment by PFF*



As it can be seen from the above labelling $s(F(2, 3, 3, 4, 5)) \leq 16$.

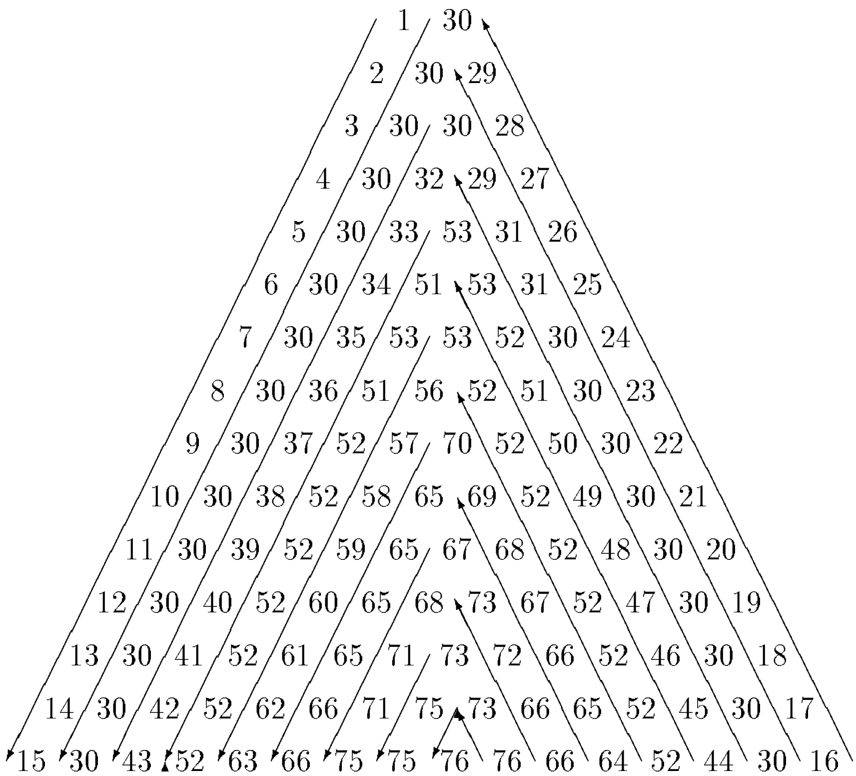
At this point we observe that the upper bound on the strength either for SFF or PFF depends on the order of the paths in the forest. PFF gives sharper upper bound over SFF when the lengths of the paths are varied. However we have an edge-labelling which gives $s(F(2, 3, 3, 4, 5)) \leq 12$ as shown below:



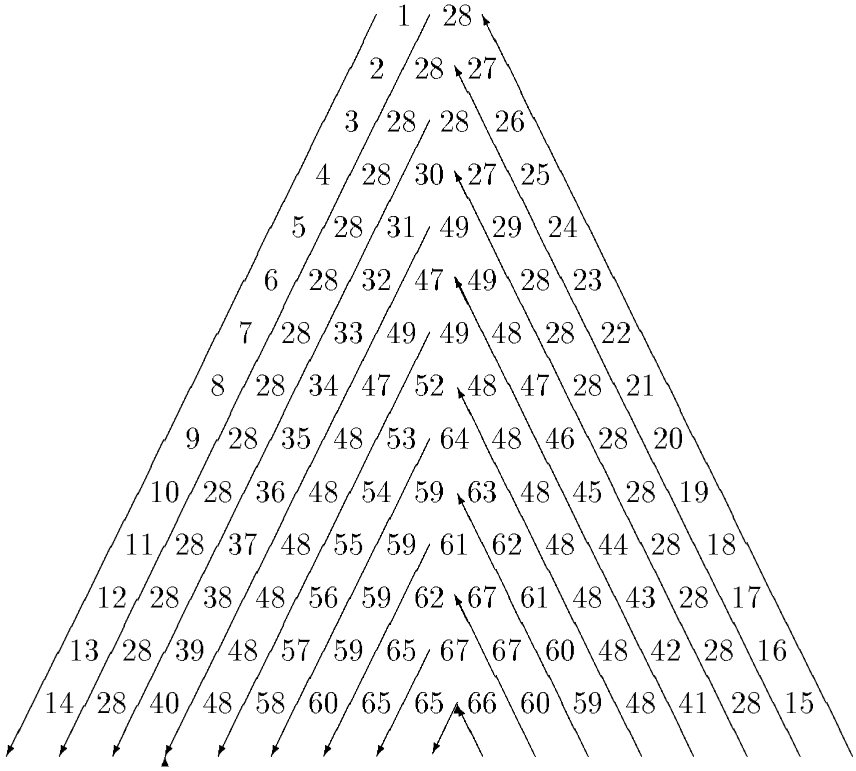
Let $F(2, 3, ..., k)$ denotes the set of disjoint paths of lengths $2, 3, ..., k$ and let $F(1, 2, 3, ..., k)$ denotes the set of disjoint paths of lengths $2, 3, ..., k$ plus with a isolate vertex. Next we will study irregular assignments and strengths of these two classes of forests. For both classes, the edges of the paths are ordered in the plane as a form of a triangle. Then we start labelling the edges from top of the triangle on the left-side edges and continue to the labelling from the bottom on the right-side edges by using the technique parallel-first-fit. And we continue labelling towards the inner shells of the triangle. Details of the constructions are omitted.

Theorem 9. *The strength of the forest* $F(2,3,...,k)$ *is* $\lceil \frac{n}{2} \rceil$ *if* $n \equiv 1 \pmod 2$ *and* $\frac{n}{2}+1$ *otherwise, where* n *is the number of the vertices of* $F(2,3,...,k)$*, i.e.,* $n = \frac{(k+2)(k-1)}{2} - 3$.

In the figure below we have shown the irregular assignments for the forests $F(2,...,16)$ and $F(2,...,15)$. In the first figure we have $s(F(2,..,16)) = 76$, where in the range $[1,152]$ only the integers 147 and 149 are not presented as vertex labels while in the second figure we have $s(F(2,...,15)) = 67$, where in the range $[1,131]$ only the integer 133 is not presented as an vertex label. Similar assignments for the forest $F(1,2,...,k)$ can also be given. Irregular assignments of the forests of paths of arbitrary lengths will be studied in a future work.



$s(F(2,3,...,16)) = 75$, missing vertex labels = 147 and 149.



$s(F(2, ..., 15)) = 67$, (missing vertex label = 133.

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