Electronic Colloquium on Computational Complexity, Comment 1 on Report No. 009 (1998)

Comment on TR98-009

B. Litow [§]

May 1, 1998

TR-98-009, *Parallel complexity of integer coprimality* contains an error in the partial fraction expansion of Eq. 3 on p.4. The correct expression is

$$S(a,b) = \frac{1}{\rho^{a+b} \cdot a \cdot b} \cdot \sum_{j=0}^{a-1} \sum_{k=0}^{b-1} \int_0^1 \frac{\omega^j \cdot \nu^k \cdot z^{-1} \cdot ds}{(z\omega^{-j} - 1/\rho)(z^{-1} \cdot \nu^{-k} - 1/\rho)}$$

The modification is essentially in the appearance of the factor $\omega^j \cdot \nu^k$ in the numerator of the integrand. The factor of $1/ab$ outside all summation is unimportant and we neglect it here.

This does not materially affect the method. Here is why. The factor $\omega^j \cdot \omega = \exp(2\pi\sqrt{-1} \cdot (j/a + k/b))$ can be approximated to the necessary precision by taking polynomially many (the parameter d) (in $\log ab$) terms in the Taylor series for exp since $|2\pi(j/a + k/b)| = O(1)$. In particular, referring to Eq. 7 on p. 5, we get

$$S_d(a,b) = \rho^{-(a+b)} \cdot \sum_{j=0}^{a-1} \sum_{k=0}^{b-1} \sum_{p=1}^{d} \sum_{q=1}^{d} G(j,k) \cdot A_{j,k,p,q}$$

where $G(j,k)$ is the polynomial in $1/j$ and $1/k$ which is the sum of the first d terms in the Taylor series for exp above. This additional factor can be summed over j and k using the same techniques used for $A_{j,k,p,q}$.

[§]Dept. of Computer Science, James Cook University, Townsville, Qld. 4811, Australia bruce@cs.jcu.edu.au

ISSN 1433-8092