

# A New Algorithm for MAX-2-SAT

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## Abstract

Recently there was a significant progress in proving (exponential-time) worst-case upper bounds for the propositional satisfiability problem (SAT). MAX-SAT is an important generalization of SAT. Several upper bounds were obtained for MAX-SAT and its NP-complete subproblems. In particular, Niedermeier and Rossmanith recently proved the worst-case upper bound  $O(2^{K/2.88\dots})$  for MAX-2-SAT (i.e. each clause contains at most two variables), where  $K$  is the number of clauses. In this paper we improve this bound to  $O(2^{K_2/4})$ , where  $K_2$  is the number of 2-clauses. In addition, our algorithm and the proof are much simpler than those of Niedermeier and Rossmanith. The key ideas are to use the symmetric flow algorithm of Yannakakis and to count only 2-clauses (and not 1-clauses).

## 1 Introduction.

*SAT* (the problem of satisfiability of a propositional formula in conjunctive normal form (*CNF*)) can be easily solved in time of the order  $2^N$ , where  $N$  is the number of variables in the input formula. In the early 1980s this trivial bound was improved for formulas in 3-CNF by Monien and Speckenmeyer [15] (see also [16]) and independently by Dantsin [3] (see also [6, 4]). After that, many upper bounds for SAT and its NP-complete subproblems were obtained ([11, 19, 22] are the most recent). Most authors consider bounds w.r.t. three main parameters: the length  $L$  of the input formula (i.e. the number of literal occurrences), the number  $K$  of its clauses and the number  $N$  of the variables occurring in it. In this paper we consider bounds w.r.t. the parameters  $K$  and  $L$ . The best such bounds for SAT are  $p(L)2^{K/3.23\dots}$  [11] ( $p$  is a polynomial) and  $O(2^{L/9.7\dots})$  (see the journal version of [11]).

The maximum satisfiability problem (*MAX-SAT*) is an important generalization of SAT. In this problem we are given a formula in CNF, and the answer is the maximal number of simultaneously satisfiable clauses. This problem is NP-complete<sup>1</sup> even if each clause contains at most two literals (*MAX-2-SAT*; see, e.g., [18]). This problem was widely studied in the context of approximation algorithms (see, e.g., [23, 9, 12, 10]). As to the worst-case time

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<sup>1</sup>A more precise NP-formulation is, of course, “given a formula in CNF, decide whether there is an assignment that satisfies at least  $k$  clauses”.

bounds for the exact solution of MAX-SAT, Niedermeier and Rossmanith [17] proved two worst-case upper bounds:  $O(L \cdot 2^{K/2.15\dots})$  for MAX-SAT<sup>2</sup>, and  $O(2^{K/2.88\dots})$  for MAX-2-SAT. For the latter bound, they presented an algorithm for MAX-SAT running in  $O(2^{L/5.76\dots})$  time<sup>3</sup>; the desired bound follows since  $L \leq 2K$ . They also posed a question whether this bound can be improved by a direct algorithm (and not by an algorithm for general MAX-SAT for a bound w.r.t.  $L$ ). In this paper, we answer this question by giving an algorithm which solves MAX-2-SAT in  $O(2^{K/4})$  time. In addition, our algorithm and the proof are much simpler.

Most of the algorithms/bounds mentioned above use the Davis-Putnam procedure [8, 7]. In short, this procedure allows to reduce the problem for a formula  $F$  to the problem for two formulas  $F[v]$  and  $F[\bar{v}]$  (where  $v$  is a propositional variable). This is called “splitting”. Before the algorithm splits each of the obtained two formulas, it can transform them into simpler formulas  $F_1$  and  $F_2$  (using some *transformation rules*). The algorithm does not split a formula if it is trivial to solve the problem for it; these formulas are the leaves of the *splitting tree* which corresponds to the execution of such algorithm. For most known algorithms, the leaves are trivial formulas (i.e. the formulas containing no non-trivial clauses).

In the algorithm presented in this paper, the leaves are satisfiable formulas and formulas for which the (polynomial time) “symmetric flow” algorithm of Yannakakis [23] finds an optimal solution (this algorithm either finds an optimal solution or simplifies the input formula). Transformation rules include the pure literal rule, a slightly generalized resolution rule (using these two rules one can solve MAX-SAT in a polynomial time in the case that each variable occurs at most twice; it was already observed in, e.g., [20]), and the frequent 1-clause rule [17]. Although in MAX-SAT 1-clauses cannot be eliminated by the usual unit propagation technique, in the case of MAX-2-SAT they can be eliminated by the symmetric flow algorithm of Yannakakis [23]. Thus, before each splitting we can transform a formula into one which consists of 2-clauses, and each variable occurs at least three times. Therefore, each splitting eliminates at least three 2-clauses in each branch. This observation would already improve the bound of [17] to  $O(2^{K_2/3})$ . However, by careful choice of a variable for splitting, we get a better bound  $O(2^{K_2/4})$ , which implies the bound  $O(2^{L/8})$  (since  $L \geq 2K_2$ ).

In Sect. 2 we give basic definitions and formulate in our framework the known results we use. In Sect. 3 we present the algorithm and the proof of its worst-case upper bound.

## 2 Background.

Let  $V$  be a set of Boolean variables. The negation of a variable  $v$  is denoted by  $\bar{v}$ . Given a set  $U$ , we denote  $\bar{U} = \{\bar{u} \mid u \in U\}$ . *Literals* (usually denoted by  $l, l', l_1, l_2, \dots$ ) are the members of the set  $W = V \cup \bar{V}$ . *Positive literals* are the members of the set  $V$ . *Negative literals* are their negations. If  $w$  denotes a negative literal  $\bar{v}$ , then  $\bar{w}$  denotes the variable  $v$ .

Algorithms for finding the exact solution of MAX-SAT are usually designed for the unweighted MAX-SAT problem. However, the formulas are usually represented by multisets

<sup>2</sup>Bansal and Raman [1] have recently improved this bound to  $O(L \cdot 2^{K/2.36\dots})$ .

<sup>3</sup>Bansal and Raman [1] have recently improved this bound to  $O(L \cdot 2^{L/6.89\dots})$  which leads to the  $O(2^{K/3.44\dots})$  bound for MAX-2-SAT.

(i.e., formulas in CNF with integer positive weights). In this paper we consider the weighted MAX-SAT problem with positive integer weights. A (*weighted*) *clause* is a pair  $(\omega, S)$  where  $\omega$  is a strictly positive integer number, and  $S$  is a nonempty finite set of literals which does not contain simultaneously any variable together with its negation. We call  $\omega$  the *weight* of a clause  $(\omega, S)$ .

An *assignment* is a finite subset of  $W$  which does not contain any variable together with its negation. Informally speaking, if an assignment  $A$  contains a literal  $l$ , it means that  $l$  has the value *True* in  $A$ . In addition to usual clauses, we allow a special *true clause*  $(\omega, \mathbb{T})$  which is satisfied by every assignment. (We also call it a  $\mathbb{T}$ -*clause*.)

The length of a clause  $(\omega, S)$  is the cardinality of  $S$ . A  $k$ -*clause* is a clause of the length exactly  $k$ . In this paper a *formula in (weighted) CNF* (or simply *formula*) is a finite set of (weighted) clauses  $(\omega, S)$ , at most one for each  $S$ . The *length of a formula* is the sum of the lengths of all its clauses. The total weight of all 2-clauses of a formula  $F$  is denoted by  $\mathfrak{K}_2(F)$ .

The pairs  $(0, S)$  are *not* clauses, however, for simplicity we write  $(0, S) \in F$  for all  $S$  and all  $F$ . Therefore, the operators  $+$  and  $-$  are defined:

$$\begin{aligned} F + G &= \{(\omega_1 + \omega_2, S) \mid (\omega_1, S) \in F \text{ and } (\omega_2, S) \in G, \text{ and } \omega_1 + \omega_2 > 0\}, \\ F - G &= \{(\omega_1 - \omega_2, S) \mid (\omega_1, S) \in F \text{ and } (\omega_2, S) \in G, \text{ and } \omega_1 - \omega_2 > 0\}. \end{aligned}$$

For a literal  $l$  and a formula  $F$ , we define

$$\begin{aligned} F[l] &= (\{(\omega, S) \mid (\omega, S) \in F \text{ and } l, \bar{l} \notin S\} + \\ &\quad \{(\omega, S \setminus \{\bar{l}\}) \mid (\omega, S) \in F \text{ and } S \neq \{\bar{l}\}, \text{ and } \bar{l} \in S\} + \\ &\quad \{(\omega, \mathbb{T}) \mid \omega \text{ is the sum of the weights } \omega' \text{ of all clauses } (\omega', S) \text{ of } F \text{ such that } l \in S\}). \end{aligned}$$

(Note that no  $(\omega, \emptyset)$  or  $(0, S)$  is included in  $F[l]$ ,  $F + G$  or  $F - G$ .) For an assignment  $A = \{l_1, \dots, l_s\}$  and a formula  $F$ , we define  $F[A] = F[l_1][l_2] \dots [l_s]$  (evidently,  $F[l][l'] = F[l'][l]$  for every literals  $l, l'$  such that  $l \neq \bar{l}'$ ). For short, we write  $F[l_1, \dots, l_s]$  instead of  $F[\{l_1, \dots, l_s\}]$ . For example, if  $F = \{(1, \{x, y\}), (5, \{\bar{y}\}), (2, \{\bar{x}, \bar{y}\}), (10, \{\bar{z}\})\}$ , then  $F[x, \bar{z}] = \{(11, \mathbb{T}), (7, \{\bar{y}\})\}$ .

The optimal value  $\text{OptVal}(F) = \max_A \{\omega \mid (\omega, \mathbb{T}) \in F[A]\}$ . An assignment  $A$  is *optimal* if  $F[A]$  contains only one clause  $(\omega, \mathbb{T})$  (or does not contain any clauses, in this case  $\omega = 0$ ) and  $\text{OptVal}(F) = \omega$  ( $= \text{OptVal}(F[A])$ ).

A formula is in *2-CNF* if it contains only 2-clauses, 1-clauses and a  $\mathbb{T}$ -clause. A formula is in *2E-CNF* if it contains only 2-clauses and a  $\mathbb{T}$ -clause.

If we say that a (positive or negative) *literal*  $v$  *occurs* in a clause or in a formula, we mean that this clause (more formally, its second component) or this formula (more formally, one of its clauses) contains the literal  $v$ . However, if we say that a *variable*  $v$  *occurs* in a clause or in a formula, we mean that this clause or this formula contains the literal  $v$ , or it contains the literal  $\bar{v}$ . A variable  $v$  *occurs positively*, if the literal  $v$  occurs, and *occurs negatively*, if the literal  $\bar{v}$  occurs. A literal  $l$  is an  $(i, j)$ -literal if  $l$  occurs exactly  $i$  times in the formula and the literal  $\bar{l}$  occurs exactly  $j$  times in the formula. A literal is *pure* in a formula  $F$  if it occurs in  $F$ , and its negation does not occur in  $F$ . The following lemma is well-known and straightforward.

**Lemma 1** *If  $l$  is a pure literal in  $F$ , then  $\text{OptVal}(F) = \text{OptVal}(F[l])$ .*

In this paper, the *resolvent*  $\mathfrak{R}(C, D)$  of clauses  $C = (\omega_1, \{l_1, l_2\})$  and  $D = (\omega_2, \{\bar{l}_1, l_3\})$  is the formula

$$\{ (\max(\omega_1, \omega_2), \mathbb{T}), (\min(\omega_1, \omega_2), \{l_2, l_3\}) \}$$

if  $l_2 \neq \bar{l}_3$ , and the formula  $\{(\omega_1 + \omega_2, \mathbb{T})\}$  otherwise. This definition is not traditional, but it is very useful in MAX-SAT context.

The following lemma is a straightforward generalization of the resolution correctness (see, e.g., [21]) for the case when there are weights, but the literal on which we are resolving does not occur in other clauses of the formula.

**Lemma 2** *If  $F$  contains clauses  $C = (\omega_1, \{v, l_1\})$  and  $D = (\omega_2, \{\bar{v}, l_2\})$  such that the variable  $v$  does not occur in other clauses of  $F$ , then*

$$\text{OptVal}(F) = \text{OptVal}( (F - \{C, D\}) + \mathfrak{R}(C, D) ).$$

The following simple observation is also well-known (see, e.g., [14, 5]).

**Lemma 3** *Let  $F$  be a formula in weighted CNF, and  $v$  be a variable. Then*

$$\text{OptVal}(F) = \max(\text{OptVal}(F[v]), \text{OptVal}(F[\bar{v}])).$$

We also note that a polynomial time algorithm for 2-SAT is known. In our context, a formula  $F$  is satisfiable if  $\text{OptVal}(F)$  is equal to the sum of the weights of all clauses occurring in  $F$ .

**Lemma 4** (see, e.g. [2]) *There is a polynomial time algorithm for 2-SAT.*

Yannakakis presented in [23] an algorithm which transforms a formula in 2-CNF into a formula in 2E-CNF which has the same optimal value. This algorithm consists of two stages. The first stage is a removal of a maximum symmetric flow from a graph corresponding to the formula; this stage can be considered as a combination of three transformation rules (it is not important for us now which combination):

1) replacing<sup>4</sup> of a “cycle”

$$\{ (\omega, \{l_1, \bar{l}_2\}), (\omega, \{l_2, \bar{l}_3\}), \dots, (\omega, \{l_k, \bar{l}_1\}) \}$$

by another cycle

$$\{ (\omega, \{\bar{l}_1, l_2\}), (\omega, \{\bar{l}_2, l_3\}), \dots, (\omega, \{\bar{l}_k, l_1\}) \};$$

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<sup>4</sup>This replacing is made by subtracting the weights: e.g., if a formula contains a clause  $(\omega', \{l_1, \bar{l}_2\})$  with  $\omega' \geq \omega$ , then it is split into two clauses  $(\omega' - \omega, \{l_1, \bar{l}_2\})$  and  $(\omega, \{l_1, \bar{l}_2\})$ , and the latter clause is replaced as formulated.

2) replacing of a set

$$\{ (\omega, \{\bar{l}_1\}), (\omega, \{l_1, \bar{l}_2\}), (\omega, \{l_2, \bar{l}_3\}), \dots, (\omega, \{l_{k-1}, \bar{l}_k\}) \}$$

by the set

$$\{ (\omega, \{\bar{l}_1, l_2\}), (\omega, \{\bar{l}_2, l_3\}), \dots, (\omega, \{\bar{l}_{k-1}, l_k\}), (\omega, \{\bar{l}_k\}) \};$$

3) replacing two contradictory clauses  $(\omega, \{l\})$  and  $(\omega, \{\bar{l}\})$  by a true clause of the weight  $\omega$ .

The second stage is replacing of the obtained formula  $F'$  by the formula  $F'[A]$  for some assignment  $A$  (it is not important for us now which assignment). Evidently, this algorithm does not increase the total weight of all 2-clauses.

**Lemma 5 ([23])** *There is a polynomial time algorithm which given an input formula  $F$  in weighted 2-CNF, outputs a formula  $G$  in weighted 2E-CNF, such that  $\mathfrak{K}_2(G) \leq \mathfrak{K}_2(F)$ , and  $\text{OptVal}(F) = \text{OptVal}(G)$ .*

The following fact was observed by Niedermeier and Rossmanith.

**Lemma 6 ([17])** *If the weight of a 1-clause  $(\omega, \{l\})$  of a formula  $F$  is not less than the total weight of all clauses of  $F$  containing the literal  $\bar{l}$ , then  $\text{OptVal}(F) = \text{OptVal}(F[l])$ .*

### 3 Results.

In this section we present Algorithm 1 which solves MAX-2-SAT in the time  $O(2^{K_2/4})$ , where  $K_2$  is the total weight of 2-clauses in the input formula (in the case of unweighted MAX-2-SAT,  $K_2$  is the number of 2-clauses).

#### Algorithm 1.

*Input:* A formula  $F$  in weighted 2-CNF.

*Output:*  $\text{OptVal}(F)$ .

*Method.*

- (1) Apply the symmetric flow algorithm from [23] (see Lemma 5) to  $F$ .
- (2) If there is a pure literal  $l$  in  $F$ , assume  $F := F[l]$ .
- (3) If there is a variable that occurs in  $F$  exactly once positively in a clause  $C$  and exactly once negatively in a clause  $D$ , then  $F := (F - \{C, D\}) + \mathfrak{R}(C, D)$ .
- (4) If  $F$  has been changed at steps (2)–(3), then go to step (1).
- (5) If  $F$  is satisfiable<sup>5</sup>, return the sum of the weights of all its clauses.

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<sup>5</sup>We can check it in a polynomial time [2], see Lemma 4.

- (6) If there is a variable  $v$  occurring in the clauses of  $F$  of the total weight at least 4, execute Algorithm 1 for the formulas  $F[v]$  and  $F[\bar{v}]$ , and return the maximum of its answers.
- (7) Find<sup>6</sup> in  $F$  a clause  $(\omega, \{l_1, l_2\})$  such that  $l_1$  and  $l_2$  are (2,1)-literals, and the two other clauses  $C$  and  $D$  containing the literals  $l_1, \bar{l}_1$  do not contain the literals  $l_2, \bar{l}_2$ . Execute Algorithm 1 for the formulas  $(F[l_1] - \{C, D\}) + \mathfrak{R}(C, D)$ , and  $F[\bar{l}_1, l_2]$ , and return the maximum of its answers.

□

**Theorem 1** *Given a formula  $F$  in 2-CNF, Algorithm 1 always correctly finds  $\text{OptVal}(F)$  in time  $O(2^{\mathfrak{R}_2(F)/4})$ .*

*Proof. Correctness.* If Algorithm 1 outputs an answer, then its correctness follows from the lemmata of Sect. 2 (step (1): Lemma 5; step (2): Lemma 1; step (3): Lemma 2; step (6): Lemma 3; step (7): Lemmata 3, 2 and 6, note that at this step  $F$  consists of the clauses of weight 1).

Since any change at steps (2) and (3) decreases the total weight of 2-clauses in  $F$ , and the step (1) does not increase it, the cycle (1)–(4) is repeated a polynomial number of times. Now it remains to show that at step (7) Algorithm 1 always can find a clause satisfying its conditions.

Note that at step (7) the formula  $F$  is not satisfiable, consists only of 2-clauses (and, maybe, a  $\mathbb{T}$ -clause), does not contain pure literals, and each variable occurs in it exactly three times, i.e.  $F$  contains only (2,1)-literals and (1,2)-literals. Since  $F$  is not satisfiable, there exists at least one clause in it that contains two (1,2)-literals (otherwise the assignment consisting of (2,1)-literals is satisfying; cf. “Extended Sign Principle” of [13]). Thus, (1,2)-literals occur in at most  $N - 1$  clauses of  $F$ , where  $N$  is the number of variables occurring in  $F$ . There are  $3N/2$  2-clauses in  $F$ . Hence,  $F$  contains more than  $N/2$  2-clauses consisting only of (2,1)-literals. There are at least  $N + 1$  literals in these clauses, thus, there is at least one (2,1)-literal occurring in *two* such clauses. This literal, and at least one of the two literals occurring with it in these clauses, satisfy the condition of the step (7).

*Running time.* Each of the steps of Algorithm 1 (not including recursive calls) takes only a polynomial time (Lemmata 5 and 4). The steps (1)–(5) do not increase the total weight of 2-clauses in  $F$ . By the above argument, each of these steps is executed a polynomial number of times during one execution of Algorithm 1 (again not including recursive calls). It suffices to show that for each formula  $F'$  which is an argument of a recursive call,  $\mathfrak{R}_2(F') \leq \mathfrak{R}_2(F) - 4$ .

Note that at the moment of a recursive call, the formula consists only of 2-clauses (and, maybe, a  $\mathbb{T}$ -clause). Then the statement follows from the conditions of the steps (6) and (7).

□

**Corollary 1** *Given a formula  $F$  in unweighted<sup>7</sup> 2-CNF of length  $L$ , Algorithm 1 always correctly finds  $\text{OptVal}(F)$  in time  $O(2^{L/8})$ .*

**Remark 1** *Of course, in Corollary 1 only the number of literal occurrences in 2-clauses is essential.*

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<sup>6</sup>Theorem 1 proves that it is possible to find a clause satisfying the conditions of this step.

<sup>7</sup>I.e., all weights equal 1.

## 4 Conclusion

In this paper we improved the existing upper bound for MAX-2-SAT with integer weights to  $O(2^{K_2/4})$ , where  $K_2$  is the total weight of 2-clauses of the input formula (or the number of 2-clauses for unweighted MAX-2-SAT). This also implies the  $O(2^{L/8})$  bound for unweighted MAX-2-SAT, where  $L$  is the number of literal occurrences (in 2-clauses).

One of the key ideas of our algorithm is to count only 2-clauses (since MAX-1-SAT instances are trivial). It would be interesting to apply this idea to SAT, for example, by counting only 3-clauses in 3-SAT (since 2-SAT instances are easy). Also, it remains a challenge to find a “less-than- $2^N$ ” algorithm for MAX-SAT or even MAX-2-SAT, where  $N$  is the number of variables.

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