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Approximating Bounded Degree Instances of NP-Hard Problems*

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Abstract

We present some of the recent results on computational complexity of approximating bounded degree combinatorial optimization problems. In particular, we present the best up to now known explicit nonapproximability bounds on the very small degree optimization problems which are of particular importance on the intermediate stages of proving approximation hardness of some other generic optimization problems.

1 Introduction

An interesting approximation hardness phenomenon of combinatorial optimization was discovered in [PY91] and [ALMSS92], to the effect that the bounded degree instances of several optimization problems are hard to approximate to within an arbitrary constant. This fact seemed to be a bit puzzling at the time as bounded degree instances of many optimization problems were known to have trivial approximation algorithms dramatically improving performances of the best known approximation algorithms on general instances. An interesting artifact on their *complementary*, i. e. dense, instances was also the existence of polynomial time approximation schemes (PTASs) for them [AKK95], [KZ97], see [K01]

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for a survey. We discuss here explicit approximation lower bounds for bounded degree instances with a very small bounds on degrees like 3 or 4, and also the best known approximation algorithms on that instances. These instances have turned out to be particularly important at the intermediate reduction stages for proving hardness of approximation of some other important optimization problems, like Set Cover, some restricted versions of Traveling Salesman Problem, and the problem of Sorting by Reversals motivated recently by molecular biology, cf. [F98], [PY91], [PY93], [BK99], [FK99], [E99], [EK00]. We mention here some interesting new results on asymptotic relations between hardness of approximation and bounds on a degree of instances [H00], [T01]. These results do not yield though explicit lower approximation bounds for small degree instances needed in applications mentioned before.

We survey in this paper the best known up to now explicit approximation lower bounds for the small degree (number of variable occurrences) optimization problems, like the problems of maximization or minimization of the satisfiability of systems of linear equations mod 2, MAX-CUT, MAX- and MIN-BISECTION, MAX-2SAT, MAXIMUM INDEPENDENT SET, and MINIMUM NODE COVER [BK99], [BK01b]. We move on, and apply these results to get explicit lower approximation bounds for the problem of Sorting by Reversals [BK99], and the Traveling Salesman Problem with distances one and two [EK00]. Finally, we mention recent improvement on approximation ratios of algorithms for small degree MAX-CUT and MAX-BISECTION problems based on local enhancing methods for semidefinite programming [FKL00a], [FKL00b], [KKL00].

2 Bounded Degree Maximization Problems

We are going to define basic optimization problems of this section.

- MAX-Ek-LIN2: Given a set of equations mod 2 with exactly k variables per equation, construct an assignment maximizing the number of equations satisfied.

- b-OCC-MAX-Ek-LIN2: Given a set of equations mod 2 with exactly k variables per equation and the number of occurrences of each variable bounded by b, construct an assignment maximizing the number of equations satisfied.

- b-OCC-MAX-HYBRID-LIN2: Given a set of equations mod 2 with exactly two or three variables per equation, and the number of occurrences of each variable bounded by b, construct an assignment maximizing the number of equations satisfied.

- b-OCC-MAX-2SAT: Given a conjunctive normal form formula with two variables per clause, construct an assignment maximizing the number of clauses satisfied.

- d-MAX-CUT: Given an undirected graph of degree bounded by d, partition its vertices into two groups so as to maximize the number of edges with exactly one endpoint in each group.

- d-MIS: Given an undirected graph of degree bounded by d, construct a maximum cardinality subset of vertices such that no two vertices of it are adjacent.

We are going to display now approximation preserving reductions which reduce from the MAX-E2-LIN2 and the MAX-E3-LIN2 problems. The method of reductions depends on a new *wheel-amplifier* construction of Berman and Karpinski [BK99] designed specially for bounded degree problems. This kind of amplifier has turned out to be more efficient than the standard expander amplifiers (cf. e.g., Arora and Lund [AL97]) for small degree, and number of occurrences, optimization problems.

We start with the following known inapproximability results of Håstad [H97].

Theorem 1. ([H97]) *For any $0 < \epsilon < \frac{1}{2}$, it is NP-hard to decide whether an instance of MAX-E2-LIN2 with 16n equations has its optimum value above* $(12-\epsilon)n$ *or below* $(11+\epsilon)$.

Theorem 2. ([H97]) *For any $0 < \epsilon < \frac{1}{2}$, it is NP-hard to decide whether an instance of MAX-E3-LIN2 with $2n$ equations has its optimum value above $(2-\epsilon)n$ or below $(1+\epsilon)n$.*

In Berman and Karpinski [BK99] the following polynomial time *randomized approximation preserving* reductions were constructed:

- f_1 : MAX-E2-LIN2 $\rightarrow$ 3-OCC-MAX-E2-LIN2,

- f_2 : MAX-E2-LIN2 $\rightarrow$ 3-MAX-CUT,

- f_3 : MAX-E3-LIN2 $\rightarrow$ 3-OCC-MAX-HYBRID-LIN2,

The constructions for f_1, and f_2 use variants of *wheel-amplifier* methods, whereas a construction for f_3 uses certain 3-hypergraph extension of it. The following *optimizing* properties of f_1, f_2, and f_3 were proven in [BK99].

Theorem 3. ([BK99]) *For any $0 < \epsilon < \frac{1}{2}$, it is NP-hard to decide whether an instance of f_1(MAX-E2-LIN2) $\in$ 3-OCC-MAX-E2-LIN2 with* 336 *edges has its optimum value above* $(332-\epsilon)n$ *or below* $(331+\epsilon)n$.

A similar result can be proven for f_2, and the 3-MAX-CUT-problem.

Theorem 4. ([BK99]) *For any* $0 < \epsilon < \frac{1}{2}$*, it is NP-hard to decide whether an instance of* f_2*(MAX-E2-LIN2)* $\in$ *3-MAX-CUT with 336 edges has its optimum value above* $(332 - \epsilon)n$ *or below* $(331 + \epsilon)n$*.*

For f_3 and MAX-HYBRID-LIN2 we have

Theorem 5. ([BK99]) *For any* $0 < \epsilon < \frac{1}{2}$*, it is NP-hard to decide whethr an instance of* f_3*(MAX-E3-LIN2)* $\in$ *3-OCC-MAX-HYBRID-LIN2 with* $60n$ *equations with exactly two variables and* $2n$ *equations with exactly three variables has its optimum value above* $(62 - \epsilon)n$ *or below* $(61 + \epsilon)n$*.*

Theorem 4 can be also used to derive the following bound for 3-OCC-MAX-2SAT.

Theorem 6. ([BK99]) *For any* $0 < \epsilon < \frac{1}{2}$*, it is NP-hard to decide whether an instance of 3-OCC-MAX-2SAT, with* $2016n$ *clauses has its optimum above* $(2012 - \epsilon)n$ *or below* $(2011 + \epsilon)n$*.*

The 3-OCC-MAX-HYBRID-LIN2 problem and Theorem 5 can be used to derive lower bounds for 4-MIS problem, and using some more subtle construction, even for 3-MIS problem.

Theorem 7. ([BK99]) *For any* $0 < \epsilon < \frac{1}{2}$*, it is NP-hard to decide whether an instance of 4-MIS with* $152n$ *nodes has its optimum value above* $(74-\epsilon)n$ *or below* $(73 + \epsilon)n$*, and whether an instance of 3-MIS with* $284n$ *nodes has its optimum value above* $(140 - \epsilon)n$ *or below* $(139 + \epsilon)n$*.*

The results above imply the following explicit nonapproximability results.

Corollary 1. *For every* $\epsilon > 0$*, it is NP-hard to approximate:*

(1) *3-OCC-MAX-E2-LIN2 and 3-MAX-CUT to within a factor* $332/331 - \epsilon$*,*

(2) *3-OCC-MAX-HYBRID-LIN2 to within a factor* $62/61 - \epsilon$*,*

(3) *3-OCC-MAX-2SAT to within a factor* $2012/2011 - \epsilon$*,*

(4) *4-MIS to within a factor* $74/73-\epsilon$*, and 3-MIS to within a factor* $140/139-\epsilon$*.*

The best to our current knowledge gaps between upper and lower approximation bounds are summarized in Table 1. The upper approximation bounds are from [GW94], [BF94], [BF95], [FG95], [FKL00a].The technical results of this section will be used also later on in our paper.

TABLE 1:
Bounded Degree Maximization Problems

Problem	Approx. Upper	Approx. Lower
3-OCC-MAX-E2-LIN2	1.0858	1.0030
3-OCC-MAX-HYBRID-LIN2	2	1.0163
3-MAX-CUT	1.0858	1.0030
3-OCC-MAX-2SAT	1.0741	1.0005
3-MIS	1.2	1.0071
4-MIS	1.4	1.0136

3 Bounded Degree Minimization Problems

We are going to introduce now the following minimization problems.

- d-Node Cover: Given an undirected graph of degree bounded by d, construct a minimum cardinality subset of vertices such that each edge of a graph has hat least one of its endpoints in it.
- MIN-Ek-LIN2: Given a set of equations mod 2 with exactly k variables per equation, construct an assignment minimizing the number of equations satisfied.
- b-OCC-MIN-Ek-LIN2: Given a set of equations mod 2 with exactly k variables per equation and the number of occurrences of each variable exactly equal to b, construct an assignment minimizing the number of equations satisfied.
- MIN-BISECTION: Given an undirected graph, partition the vertices into two equal halves so as to minimize the number of edges with exactly one endpoint in each half.
- d-MIN-BISECTION: Given a d-regular graph, partition the vertices into two equal halves so as to minimize the number of edges with exactly one endpoint in each half.

We will specialize now techniques of Section 2 to obtain lower approximation bounds on bounded degree minimization problems.

We start with a direct application of Theorem 7 towards d-Node Cover problem. For a given undirected graph $G = (V, E)$, and a *maximum independent* set I of G, $V \backslash I$ is a *minimum node cover* of G. We take now an instance of 4-MIS with $152n$ nodes. It is NP-hard, for any $0 < \epsilon < \frac{1}{2}$, to decide whether 4-Node Cover has its optimum value above $(152 - 73 - \epsilon)n = (79 - \epsilon)n$ or below $(152 - 74 + \epsilon)n = (78 + \epsilon)n$. Similarly for 3-Node Cover. Thus we have

Theorem 8. *For any $0 < \epsilon < \frac{1}{2}$, it is NP-hard to decide whether an instance of 4-Node Cover with $152n$ nodes has its optimum value above $(79 - \epsilon)n$ or below $(78 + \epsilon)n$, and whether an instance of 3-Node Cover with $284n$ has its optimum value above $(145 - \epsilon)n$ or below $(144 + \epsilon)n$.*

Theorem 8 gives the following approximation lower bounds for 4-Node Cover and 3-Node Cover problems.

Corollary 2. *For every $\epsilon > 0$, it is NP-hard to approximate*

1. *3-Node Cover to within a factor $145/144 - \epsilon$,*
2. *4-Node Cover to within a factor $79/78 - \epsilon$.*

We turn now to the bounded occurrence minimum satisfiability of linear equations.

We need the following recent result of Dinur, Kindler, Raz and Safra [DKRS00] (see also [DKS98], [KST97]).

Theorem 9.([DKRS00]) *MIN-LIN2 is NP-hard to approximate to within a factor $n^{c/loglogn}$ for some constant c.*

MIN-LIN2 is equivalent to the well known Nearest Codeword problem (cf. [ABSS93]). Only very recently the first sublinear approximation ratio $O(n/logn)$ algorithm was designed by Berman and Karpinski [BK01b].

We introduce now a notion of an (r, t)-approximation algorithm. For two functions r and t, we call an approximation algorithm A for an optimization problem P, an $(r(n), t(n))$-approximation algorithm if A approximates P within an approximation ratio $r(n)$ and A works in $O(t(n))$ time for n a size of an instance.

Berman and Karpinski [BK01b] proved the following result on the (r, t)-approximations of the 3-OCC-MIN-E3-LIN2 problem.

Theorem 10. ([BK01b]) *There exists a constant c such that if there exists an $(r(n), t(n))$-approximation algorithm for 3-OCC-MIN-E3-LIN2, then there exists an $(r(cn), t(cn))$-approximation algorithm for MIN-LIN2.*

Theorem 9 entails now

Theorem 11. *The problem 3-OCC-E3-LIN2 is NP-hard to approximate to within a factor $n^{c/loglogn}$ for some constant c.*

The 3-OCC-MIN-E3-LIN2 problem is equivalent to the exactly-3 bounded occurrence 3-ary Nearest Codeword problem (c.f. [KST97]), and therefore we have

Corollary 3. *The 3-ary Nearest Codeword problem with the number of occurrences of each variable exactly equal to 3 is NP-hard to approximate to within a factor $n^{c/loglogn}$ for some constant c.*

We apply a similar technique for the problem of MIN-BISECTION. Here our result will be only relative to the approximation hardness of MIN-BISECTION, the status of which is wide open, and we know currently of no proof technique which excludes existence of a PTAS for that problem.

Somewhat surprisingly in that context, Berman and Karpinski [BK01b] proved the following result on approximation hardness of bounded degree version of MIN-BISECTION.

Theorem 12. ([BK01b]) *If there exists an $(r(n), t(n))$-approximation algorithm for 3-MIN-BISECTION, then there exists an $(r(n^3), t(n^3))$- approximation algorithm for MIN-BISECTION.*

The best currently known approximation algorithm for the MIN-BISECTION is of ratio $O(log^2 n)$ due to Feige and Krauthgamer [FK00]. Any asymptotic improvement on approximation ratio r for 3-regular graphs, say $r = o(log^2 n)$, will entail, by Theorem 12, an improvement on an approximation ratio for the general MIN-BISECTION.

A similar technique can be also used to prove approximation hardness result for the general planar MIN-BISECTION of the planar MIN-BISECTION problem on 3-regular graphs.

4 Some Application

We are going to apply our previous results for some other generic optimization problems. The first problem is one of the most important problems in analysis of genome rearrangements, and it is being recently also motivated by other algorithmic problems of computational molecular biology.

- MIN-SBR (Sorting by Reversals): Given a permutation, construct a minimum length sequence of reversals (see for definitions [BP96]) which transforms it to the identity permutation.

We refer also to some other variants of Sorting by Reversals problems studied in [C99], called MSBR and Tree SBR (see the definitions there).

The proof technique used in [BK99] to prove explicit approximation lower bound of Theorem 7 for 4-MIS can be adapted to prove for the first time the inapproximability of MIN-SBR, and, in fact, also giving an explicit approximation lower bound for that problem.

Theorem 13. ([BK99]) *For every* $\epsilon > 0$*, it is NP-hard to approximate MIN-SBR within a factor* $1237/1236 - \epsilon$*.*

Caprara [C99] has used the above result to prove inapproximability of the both beforementioned problems, MSBR, and Tree SBR, and to compute the first explicit approximation lower bounds for those problems.

We turn now to another application of the results of Section 2. We denote by (1,2)-TSP the Traveling Salesman Problem with distances one and two, and its asymmetric version by (1,2)-ATSP (cf. [PY93], [V92]).

Engebretsen and Karpinski [EK00] has used recently the result on 3-OCC-MAX-HYBRID-LIN2 of Theorem 5 to prove the following explicit inapproximability result for (1,2)-ATSP problem.

Theorem 14. ([EK00]) *For every* $\epsilon > 0$*, it is NP-hard to approximate (1,2)-ATSP within a factor* $321/320 - \epsilon$*.*

The construction used by Engebretsen and Karpinski [EK00] could be also adapted to yield an explicit result for (1,2)-TSP.

Theorem 15. ([EK00]) *For every* $\epsilon > 0$*, it is NP-hard to approximate (1,2)-TSP within a factor* $743/742 - \epsilon$*.*

5 New Upper Approximation Bounds

The intricacy of proving the first explicit approximation lower bounds for small degree optimization problems, and the resulting huge gaps between upper and lower approximation bounds has stimulated research on improving approximation ratios for those problems as well as for some other generic problems.

The first gap for 3-MAX-CUT (and 3-OCC-MAX-E2-LIN2) was improved recently by Feige, Karpinski and Langberg [FKL00a], see Table 1. The technique of [FKL00a] is based on a new local enhancing method for semidefinite programs.

Theorem 16. ([FKL00a]) *There exists a polynomial time algorithm approximating 3-MAX-CUT within a factor 1.0858.*

We note that the best approximation ratio currently known for MAX-CUT problem on general graphs is 1.1383 ([GW94]), and the best known approximation lower bound is 1.0624 [H97]. We note also that for the semidefinite relaxation of MAX-CUT used in [GW94], the integrality gap is at least 1.1312, even for 2-regular graphs. Thus the bound of Theorem 16 beats the integrality bound even for 2-regular graphs.

We turn now to the special case of regular bounded degree graphs, and will investigate approximation algorithms for the MAX-CUT and MAX-BISECTION (partitioning of a graph into two halves so as to maximize a number of the edges between them).

Rd-MAX-CUT and Rd-MAX-BISECTION are the MAX-CUT and MAX-BISECTION problems, respectively, restricted to d-regular graphs.

Feige, Karpinski and Langberg [FKL00a], [FKL00b] were able to improve the best known approximation ratios for both bounded degree problems, Rd-MAX-CUT, and Rd-MAX-BISECTION. The best known approximation ratio for MAX-BISECTION on general graphs is 1.4266 [HZ00].

Theorem 17. ([FKL00a], [FKL00b]) *There are polynomial time algorithms that approximate R3-MAX-CUT and R3-MAX-BISECTION problems within factor 1.0823 and 1.1991, respectively.*

Using an additional local adhancement method, Karpinski, Kowaluk and Lingas [KKL00], have further improved approximation ratios of the low degree Rd-MAX-BISECTION problems.

Theorem 18. ([KKL00]) *There exists a polynomial time algorithm approximating R3-MAX-BISECTION within a factor 1.1806.*

Interestingly, the first improvements on approximation ratios of MAX-BISECTION on low degree planar graphs undertaken in [KKL00] has lead to design of the first PTASs for the general planar MAX-BISECTION as well as for other geometrically defined classes of graphs (see [JKLS01]).

On the lower bounds side, we note that the techniques of [BK99] yield also the best up to now explicit approximation lower bounds for R3-MAX-CUT, and R3-MAX-BISECTION problems equal to the lower approximation bound for 3-MAX-CUT problem of Section 2.

6 Summary of Approximation Results on Bounded Degree Minimization Problems

We present here (Table 2) the results of Section 3 and 4 on bounded degree minimization problems and the best to our knowledge gaps between upper and lower approximation bounds on those problems. The upper approximation bounds are from [BF94], [BF95], [BK01b], [FK00], [BHK01], [V92], [PY93].

TABLE 2:
Bounded Degree and Weight Minimization Problems

Problem	Approx. Upper	Approx. Lower
3-Node Cover	1.1666	1.0069
4-Node Cover	1.2857	1.0128
3-OCC-MIN-E3-LIN2	$O(\mathrm{n}/\mathrm{logn})$	$\mathrm{n}^{\Omega(1)/\mathrm{loglogn}}$
3-MIN-BISECTION	$O(\log^2 \mathrm{n})$	Equal to MIN-BISECTION
MIN-SBR	1.375	1.0008
(1,2)-TSP	1.1667	1.0013
(1,2)-ATSP	1.4167	1.0031

7 Open Problems and Further Research

An obvious open problem is to improve on both the lower and upper approximation bounds of bounded degree optimization problems, especially on those with the very small degree bounds. The essential improvements on the explicit lower bounds for these problems might be of paramount difficulty though, but same time they are also of great interest. Any such improvement would have immediate effects on the explicit lower bounds for other optimization problems, as indicated in this paper. Perhaps somewhat easier undertaking would be improving on upper approximation bounds. Here essential improvements were already achieved on the problems like a small degree MAX-CUT, and MAX-BISECTION mentioned in Section 5. How about improvements on other bounded degree optimization problems?

References

[ABSS93] S. Arora, L. Babai, J. Stern and Z. Sweedyk, *The Hardness of Approximate Optima in Lattice, Codes, and Systems of Linear Equations*, Proc. of 34th IEEE FOCS, 1993, 724-733.

[AKK95] S. Arora, D. Karger, and M. Karpinski, *Polynomial Time Approximation Schemes for Dense Instances of NP-Hard Problems*, Proc. 27th ACM STOC (1995), pp. 284-293; the full version appeared in J. Comput. System Sciences 58 (1999), pp. 193-210.

[AL97] S. Arora and C. Lund, *Hardness of Approximations*, in *Approximation Algorithms for NP-Hard Problems* (D. Hochbaum, ed.), PWS Publ. Co. (1997), pp. 399-446.

[ALMSS92] S. Arora, C. Lund, R. Motwani, M. Sudan and M. Szegedy, *Proof Verification and Hardness of Approximation Problems*, Proc. 33rd IEEE FOCS (1992), pp. 14-20.

[BF94] P. Berman and M. Fürer, *Approximating Maximum Independent Set in Bounded Degree Graphs*, Proc. 5th ACM-SIAM SODA (1994), pp. 365-371.

[BF95] P. Berman and T. Fujito, *Approximating independent sets in degree 3 graphs*, Proc. 4th Workshop on Algorithms and Data Structures, LNCS Vol. 955, Springer-Verlag, 1995, pp. 449-460.

[BHK01] P. Berman, S. Hannenhalli and Karpinski, *1.375-Approximation Algorithm for Sorting by Reversals*, Manuscript, 2001.

[BK99] P. Berman and M. Karpinski, *On Some Tighter Inapproximability Results*, Proc. 26th ICALP (1999), LNCS 1644, Springer, 1999, pp. 200-209.

[BK01a] P. Berman and M. Karpinski, *Approximating Minimum Unsatisfiability of Linear Equations*, ECCC Technical Report TR01-025 (2001).

[BK01b] P. Berman and M. Karpinski, *Approximation Hardness of Bounded Degree MIN-CSP and MIN-BISECTION*, ECCC Technical Report TR01-026 (2001).

[BP96] V. Bafna and P. Pevzner, *Genome Rearrangements and Sorting by Reversals*, SIAM J. on Computing 25 (1996), pp. 272-289.

[C99] A. Caprara, *Formulations and Hardness of Multiple Sorting by Reversals*, Proc. ACM RECOMB'99, pp. 84-93.

[DKS98] I. Dinur, G. Kindler and S. Safra, *Approximating CVP to Within Almost Polynomial Factors is NP-hard*, Proc. of 39th IEEE FOCS, 1998, 99-109.

[DKRS00] I. Dinur, G. Kindler, R. Raz and S. Safra, *An Improved Lower Bound for Approximating CVP*, 2000, submitted.

[E99] L. Engebretsen, *An Explicit Lower Bound for TSP with Distances One and Two*, Proc. 16th STACS (1999), LNCS 1563 (1999), Springer, 1999, pp. 371-382.

[EK00] L. Engebretsen and M. Karpinski, *Approximation Hardness of TSP with Bounded Metrics*, ECCC Technical Report TR00-089 (2000), to appear in Proc. 28th ICALP (2001).

[F98] U. Feige, *A Threshold of ln n for Approximation Set Cover*, J. of ACM **45** (1998), pp. 634-652.

[FG95] U. Feige and M. Goemans, *Approximating the Value of Two Prover Proof Systems with Applications to MAX-2SAT and MAX-DICUT*, Proc. 3rd Israel Symp. on Theory of Computing and Systems, 1995, pp. 182-189.

[FK00] U. Feige and R. Krauthgamer, *A Polylogarithmic Approximation of the Minimum Bisection*, Proc. 41st IEEE FOCS (2000), pp. 105 - 115.

[FKL00a] U. Feige, M. Karpinski, and M. Langberg, *Improved Approximation of MAX-CUT on Graphs of Bounded Degree*, ECCC Technical Report TR00-021 (2000), submitted to J. of Algorithms.

[FKL00b] U. Feige, M. Karpinski, and M. Langberg, *A Note on Approximation MAX-BISECTION on Regular Graphs*, ECCC Technical Report TR00-043 (2000), to appear in Information Processing Letters.

[FK99] W. Fernandez de la Vega and M. Karpinski, *On Approximation Hardness of Dense TSP and Other Path Problems*, Information Processing Letters 70 (1999), pp. 53-55.

[GW94] M. Goemans and D. Williamson, *.878-approximation Algorithms for MAX-CUT and MAX2SAT*, Proc. 26th ACM STOC (1994), pp. 422-431.

[H97] J. Håstad, *Some Optimal Inapproximability Results*, Proc. 29th ACM STOC (1997), pp. 1-10.

[H00] J. Håstad, *On Bounded Occurrence Constraint Satisfaction*, Information Processing Letters 74 (2000), pp. 1-6.

[HZ00] E. Halperin and U. Zwick, *Improved Approximation Algorithms for Maximum Graph Bisection Problems*, Manuscript, 2000.

[JKLS01] K. Jansen, M. Karpinski, A. Lingas, and E. Seidel, *Polynomial Time Approximation Schemes for MAX-BISECTION on Planar and Geometric Graphs*, Proc. 18th STACS (2001), LNCS 2010, Springer, 2001, pp. 365-375.

[K01] M. Karpinski, *Polynomial Time Approximation Schemes for Some Dense Instances of NP-Hard Optimization Problems*, Algorithmica **30** (2001), pp. 386-397.

[KKL00] M. Karpinski, M. Kowaluk, and A. Lingas, *Approximation Algorithms for MAX-BISECTION on Low Degree Regular Graphs and Planar Graphs*, ECCC Technical Report TR00-051 (2000).

[KZ97] M. Karpinski and A. Zelikovsky, *Approximating Dense Cases of Covering Problems*, ECCC Technical Report TR 97-004, 1997, also in Proc. DIMACS Workshop on Network Design: Connectivity and Facilities Location, Princeton, 1997, DIMACS Series in Discrete Mathematics and Theoretical Computer Science 40 (1998), pp. 169-178.

[KST97] S. Khanna, M. Sudan and L. Trevisan, *Constraint Satisfaction: the approximability of minimization problems*, Proc. of 12th IEEE Computational Complexity, 1997, 282-296.

[PY91] C. H. Papadimitriou and M. Yannakakis, *Optimization, Approximation and Complexity Classes*, J. Comput. System Sciences 43 (1991), pp. 425-440.

[PY93] C. H. Papadimitriou and M. Yannakakis, *The Traveling Salesman Problem with Distances One and Two*, Math. of Oper. Res. 18 (1993), pp. 1-11.

[T01] L.Trevisan, *Non-approximability Results for Optimization Problems on Bounded Degree Instances*, to appear in Proc. 33rd ACM STOC (2001).

[V92] S. Virhwanathan, *An Approximation Algorithm for the Asymetric Travelling Salesman Problem with Distances One and Two*, Information Processing Letters 44 (1992), pp. 297-302.

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