Electronic Colloquium on Computational Complexity, Report No. 39 (2004)

On approximation of the maximum clique minor containment problem and some subgraph homeomorphism problems

Andrzej Lingas[1] and Martin Wahlen[1]

Department of Computer Science, Lund University, 22100 Lund.
Andrzej.Lingas@cs.lth.se, martin@cs.lth.se, Fax +46 46 13 10 21

Abstract. We consider the "minor" and "homeomorphic" analogues of the maximum clique problem, i.e., the problems of determining the largest h such that the input graph has a minor isomorphic to K_h or a subgraph homeomorphic to K_h, respectively. We show the former to be approximable within $O(\sqrt{n}\log^{1.5} n)$ by exploiting the minor separator theorem of Plotkin *et al.* Next, we show an $\Omega(n^{1/2-O(1/(\log n)^{\gamma})})$ lower bound (for some constant γ, unless $\mathcal{NP} \subseteq \mathrm{ZPTIME}(2^{(\log n)^{O(1)}})$), and an $O(n \log\log n/\log^{1.5} n)$ upper bound on the approximation factor for maximum homeomorphic clique. Finally, we study the problem of subgraph homeomorphism where the guest graph has maximum degree not exceeding three and low treewidth. In particular, we show that for any graph G on n vertices and a positive integer q not exceeding n, one can produce either n/q approximation to the longest cycle problem and $(n-1)/(q-1)$ approximation to the longest path problem, both in polynomial time, or a longest cycle and a longest path of G in time $2^{O(q\sqrt{n}\log^{2.5} n)}$.

1 Introduction

Considered as an injective mapping, the *subgraph isomorphism* of P into G consists of a mapping of vertices of P into vertices of G so that edges of P map to corresponding edges of G. Generalizations of this mapping include *subgraph homeomorphism*, or equivalently, *topological embedding*, where vertices of P map to vertices of G and edges of P map to vertex-disjoint paths in G, and *minor containment*, where vertices of P map to disjoint connected subgraphs of G and edges of G map to edges of G.

All these problems are inherently NP-complete when the pattern or guest graph P are not fixed [12]. For fixed P, all are solvable in polynomial time, which in case of subgraph homeomorphism and minor containment is highly non-trivial to show [23]. They remain to be NP-complete for several special graph classes, e.g., for graphs of bounded treewidth [14, 20]. Restricting the pattern graph P to complete graphs or simple cycles or paths does not help in the case of subgraph isomorphism. The maximum clique, Hamiltonian cycle and Hamiltonian path problems are well known as basic NP-complete problems [12]. Their optimization versions are also known to be very difficult to approximate. For instance,

ISSN 1433-8092

it is known that unless $\mathcal{NP} \subseteq$ ZPTIME$(2^{(\log n)^{O(1)}})$, no polynomial-time algorithm for maximum clique (or, equivalently, for maximum independent set) can achieve the approximation factor of $n^{1-O(1/(\log n)^{\gamma})}$ for some constant γ [19] (see also [7, 15]). On the other hand, the best known polynomial-time approximation algorithm for maximum clique achieves solely an $n \log^2 \log n / \log^3 n$ factor [10]. The situation is not better in case of the optimization versions of the Hamiltonian cycle and path problems, called the *longest cycle* and *longest path* problems [9]. For example, the best known polynomial-time approximation algorithm for the longest path problem achieves only $n \log\log n/ \log^2 n$ factor [3, 11] [1]. The longest path problem cannot be approximated within any constant factor in polynomial time unless $P = NP$ or within any $2^{O(\log^{1-\epsilon} n)}$ factor, where $\epsilon > 0$, in polynomial time, unless $NP \subset DTIME(2^{\log^{1/\epsilon} n})$ [18]. Generally, the directed versions of the longest cycle and longest path problems seem to be even harder (see [3]). Nevertheless, on the positive side, in graphs of maximum degree not exceeding three it is possible to approximate the longest cycle problem within $O(n^{1-(\log_3 2)/2})$ in polynomial time [9]. Furthermore, it is shown in [11] that a path of length k (if it exists) can be found in time $2^{O(k)} n^{O(1)}$ which implies that the longest path problem is fixed-parameter tractable.

In the first part of this paper, we consider the "minor" analogue of the maximum clique problem, i.e., the problem of determining the largest h such that the input graph has a minor isomorphic to K_h. We show that the minor separator theorem of Plotkin *et al.* [21] yields an approximation algorithm which for a graph on n vertices and m edges produces a minor isomorphic to K_q, where $q = \Omega(h/(\sqrt{n} \log^{1.5} n))$, in time $O(mn \log^{1.5} n)$.

In the second part, we consider the maximum homeomorphic clique problem, i.e., the problem of determining the largest h such that the input graph has a subgraph homeomorphic to K_h. We show that the aforementioned results on the approximability of the standard maximum clique problem yield an $\Omega(n^{1/2-O(1/(\log n)^{\gamma})})$ lower bound for some constant γ, unless $\mathcal{NP} \subseteq$ ZPTIME$(2^{(\log n)^{O(1)}})$, and an $O(n(\log\log n)/\log^{1.5} n)$ upper bound on the approximation factor for maximum homeomorphic clique.

Our results give evidence that the maximum clique minor containment problem might be somewhat easier than the subgraph isomorphism and homeomorphism problems. The spectacular result of Robertson and Seymour [23] showing that for any fixed guest graph the minor containment problem is solvable in cubic time implies that it is so called *fixed-parameter tractable* [6]. The maximum clique problem is complete for the so called class $W[1]$ (see [6]). Hence, the maximum clique problem as well as its generalization, the subgraph isomorphism problem, are likely to be fixed-parameter intractable. (On the other hand, the subgraph isomorphism and homeomorphism problems restricted to k-connected partial k-trees are solvable in polynomial time [20, 13] whereas the minor containment problem is still NP-complete under such restriction [20].)

[1] In the forthcoming STOC 2004 paper, Gabow derives an $n/\exp(\Omega(\sqrt{\log n/ \log\log n}))$ approximation factor for the longest path problem by iterating the method of Björklund and Husfeldt from [3].

In the third part, we study the subgraph homeomorphism (equivalently, topological embedding) problem for guest graphs of maximum degree not exceeding three and low treewidth. Note that a path or a cycle belongs to the aforementioned class of graphs. Again relying on the minor separator theorem of Plotkin *et al.* [21], we obtain among other things the following partial result on the approximability and time complexity of the longest cycle and path problems:
For a graph G on n vertices, and a positive integer q smaller than n, one can produce either a simple cycle in G of length not less than q in polynomial time, thus yielding n/q polynomial-time approximation to the longest cycle problem and $(n-1)/(q-1)$ polynomial-time approximation to the longest path problem, or a longest cycle and a longest path of G in time $2^{O(q\sqrt{n}\log^{2.5} n)}$. For instance, if we set q to $\lfloor n^{1/4} \log^{-1.25} n \rfloor$ then we obtain either about $n^{3/4} \log^{1.25} n$ approximation guarantee in polynomial time or optimal solutions in subexponential time $2^{O(n^{3/4} \log^{1.25} n)}$ for both problems.
Of course, the practical usefulness of our partial result is limited since the potential user cannot choose between these two possibilities. However, this result suggests that perhaps at least one of these possibilities may hold in general. Presently no subexponential algorithms for the longest cycle or path problems are known, e.g., the fastest known algorithm for Hamiltonian cycle in cubic graphs runs in time $O(2^{n/3})$ [8]. Hence, both proving the existence of an $n^{1-\epsilon}$ polynomial-time approximation to the longest cycle and path problems as well as proving the existence of subexponential algorithm for these problems would be surprising and spectacular results (see [17]).

2 Preliminaries

We begin with a formal definition of a (balanced) separator of a graph.

Definition 1. *A* b-separator *of a graph on n vertices and m edges is a subset X of the vertex set of G whose removal from the graph splits it into connected components, none of which has more than b fraction of the sum the weights of the vertices and edges. The size of the separator is $|X|$. If not otherwise stated we shall assume the vertices to have weight* 1 *and the edges to have weight* 0.

Let k be a positive integer. A graph G on n vertices is said to be k-separable *if either it has at most $k+1$ vertices or it has a $\frac{2}{3}$-separator of size at most k whose removal splits G into two k-separable subgraphs.*

We shall denote the complete graph on q vertices by K_q and if a graph G has a minor isomorphic to a graph P, say that G has a P-minor.

The minor separator theorem due to Alon *et al.* [1] can be formulated as follows.

Fact 1 [1]. *There is an algorithm that for a graph on n vertices and m edges, and an integer q, either produces its K_q-minor or finds its $\frac{2}{3}$-separator of size $O(q^{3/2}\sqrt{n})$ in time $O(\sqrt{qn}(n+m))$.*

Fact 1 has been improved by Plotkin *et al.* in [21] as follows.

Fact 2 [21]. *There is an algorithm that for a graph on n vertices and m edges, and an integer q, either produces its K_q-minor or finds its $\frac{2}{3}$-separator of size $O(q\sqrt{n \log n})$ in time $O(m\sqrt{n \log n})$.*

The notion of *treewidth* of a graph was originally introduced by Robertson and Seymour [22] as one of the main contributions in their seminal graph minor project. It has turned out to be equivalent to several other interesting graph theoretic notions, e.g., the so called partial k-trees (e.g., see [2, 4]).

Definition 2. *A* tree decomposition *of a graph $G = (V, E)$ is a pair $(\{X_i \mid i \in I\}, T = (I, F))$, where $\{X_i \mid i \in I\}$ is a collection of subsets of V, and $T = (I, F)$ is a tree, such that the following conditions hold:*

1. $\bigcup_{i \in I} X_i = V$,
2. *for all edges $(v, w) \in E$, there exists a node $i \in I$, with $v, w \in X_i$, and*
3. *for every vertex $v \in V$, the subgraph of T, induced by the nodes $\{i \in I \mid v \in X_i\}$ is connected.*

Each set X_i, $i \in I$, is called the bag *associated with the ith node of the decomposition tree T. The* width *of a tree decomposition $(\{X_i \mid i \in I\}, T = (I, F))$ is $\max_{i \in I} |X_i| - 1$. The* treewidth *of a graph is the minimum width of its tree decomposition taken over all possible tree-decompositions of the graph. A* path decomposition *of a graph, the width of a path decomposition and the* pathwidth *of a graph are defined analogously by constraining T to be just a path.*

The following fact follows from the proof of Theorem 20 in [4].

Fact 3 *Let G be a graph on n vertices. If a sequence of sets on at most l vertices in G satisfying the requirements for the $\frac{2}{3}$-separators in the definition of l-separability of G (see Def. 1) is given then a path decomposition of G of width $O(l \log n)$ can be computed in time $O(nl \log n)$.*

Proof. Sketch. Let S be the indicated splitting set, and let G_1 and G_2 be the two subgraphs resulting from removing S. Recursively construct path decompositions $(X_1, ..., X_r)$ and $(Y_1, ..., Y_q)$ for G_1 and G_2, respectively. Then, form the path decomposition $(X_1 \cup S, ..., X_r \cup S, Y_1 \cup S, ..., Y_q \cup S)$ for G.

Lemma 16 in [4] yields the next fact.

Fact 4 *If H is a minor of G then the treewidth of H does not exceed the treewidth of G and the pathwidth of H does not exceed the pathwidth of G.*

Theorem 5.2 in [13] yields the following fact.

Fact 5 *Let P and G be graphs of treewidth l, on n vertices totally, and let the maximum degree in P be $O(1)$. One can determine whether or not P can be topologically embedded in G, and if so, produce a topological embedding of P in G in time $O(n^{l+2})$.*

3 Approximation of maximum K_q-minor

It is well known that given a tree decomposition T of a graph G, for any clique in G there is a bag of T wholly including it (see Lemma 4 in [4] and [5]). Hence, the treewidth of K_q is not smaller than $q-1$. Combing this with Fact 4, we obtain immediately the following useful lemma.

Lemma 1. *If a graph G has a tree decomposition or a path decomposition of width l then the largest integer h such that G has a K_h-minor does not exceed $l+1$.*

By Lemma 1, we obtain the following key lemma.

Lemma 2. *There is an algorithm that for a graph G on n vertices either produces its path decomposition of width $O(q\sqrt{n}\log^{1.5} n)$ or its minor isomorphic to K_q in time $O(mn^{1.5}\sqrt{\log n})$.*

Proof. Run the algorithm of Plotkin *et al.* from Fact 2 for K_q and G or K_q and a subgraph of G to produce a $\frac{2}{3}$-separator of G, or of a subgraph of G, respectively, having $O(q\sqrt{n\log n})$ vertices, in order to obtain a path decomposition of G having width $O(q\sqrt{n}\log^{1.5} n)$ by Fact 3. We may assume w.l.o.g that the algorithm never fails to produce the aforementioned separator since otherwise we obtain a minor of G isomorphic to K_q. More exactly, given such a separator, we remove it from the current subgraph of G in order to compute the resulting connected components and group them in two sub-subgraphs, none containing more than two thirds of the vertices of the current subgraph, and then run the algorithm of Plotkin *et al.* on these two sub-subgraphs and so on. By Fact 3, such a sequence of separators yields a path decomposition of width $O(q\sqrt{n}\log^{1.5} n)$. To obtain the time bound it is sufficient to observe that the algorithm of Plotkin *et al.* is run at most $n-1$ times on G and its subgraphs and that we may assume w.l.o.g $m \geq n-1$.

We show our first main result on maximum clique minor containment by forcing the algorithm of Plotkin *et al.* from Fact 2 to produce K_q-minors for sufficiently small q.

Theorem 1. *Let G be a graph on n vertices and m edges, and let h be the largest integer h such that G has a K_h-minor. There is an $O(mn^{1.5}\log^{1.5} n)$-time algorithm which determines a minor of G isomorphic to K_q where $q = \Omega(h/\sqrt{n}\log^{1.5} n)$.*

Proof. Let l be the minimum width of a path decomposition of G. By Lemma 2 there is a constant c_1 such that if q was a positive integer smaller than $c_1 l/\sqrt{n}\log^{1.5} n$ and G had no K_q minor then G would have a path decomposition smaller than l. Let q be a positive integer smaller than $c_1 l/\sqrt{n}\log^{1.5} n$. Since G cannot have a path decomposition of width smaller than l, the algorithm from Lemma 2 has to produce a minor of G isomorphic to K_q. Let $[1,r]$ be the maximal interval such that for any integer $q \in [1,r]$, the algorithm from

Lemma 2 applied to G produces a minor of G isomorphic to K_q. It follows that $r \geq c_1 l/\sqrt{n}\log^{1.5} n - 1$. We can find a positive integer p not smaller than r such that the algorithm from Lemma 2 applied to G produces a minor of G isomorphic to K_p by performing binary search, running logarithmic number of times this algorithm. It takes $O(mn^{1.5}\log^{1.5} n)$ time totally by Lemma 2. We have $p \geq c_1 l/\sqrt{n}\log^{1.5} n - 1$. On the other hand, we have $h = O(l)$ by Lemma 1. Thus, there is a constant c_2 such that $p \geq c_2 h/\sqrt{n}\log^{1.5} n$.

4 Approximability of maximum homeomorphic clique

The following lemma will be useful in proving our lower and upper bounds on approximability of maximum homeomorphic clique.

Lemma 3. *There is an algorithm which for a homeomorphic clique of size h in a graph on n vertices determines a clique of size $\Omega(h^2/n)$, contained in the homeomorphic clique, in time polynomial in n.*

Proof. Let h be the number of clique vertices, i.e., endpoints of paths modeling clique edges, in a homeomorphic clique $\tilde{H}$. Note that $\tilde{H}$ can include at most $n-h$ paths having more than one edge directly connecting its clique vertices. Form an auxiliary graph A on the clique vertices of $\tilde{H}$ such that two vertices u and v are connected by an edge if and only if the shortest path in $\tilde{H}$ connecting them has length at least two. Note that A has at most $n-h$ edges and consequently average degree $(n-h)/2h$. Hence, by [16], one can determine an independent set of size $\Omega(h^2/n)$ in A and consequently a clique of size $\Omega(h^2/n)$ in $\tilde{H}$, in polynomial time.

Our lower bound on polynomial-time approximability of maximum homeomorphic clique follows from that for maximum clique [19] (see also [7, 15]) by Lemma 3.

Theorem 2. *Unless $\mathcal{NP} \subseteq ZPTIME(2^{(\log n)^{O(1)}})$, maximum homeomorphic clique cannot be approximated within a factor $n^{1/2-O(1/(\log n)^{\gamma})}$ for some constant γ.*

Proof. By [19], no polynomial-time algorithm for maximum clique can achieve the approximation factor of $n^{1-O(1/(\log n)^{\gamma})}$ for some constant γ unless $\mathcal{NP} \subseteq$ ZPTIME$(2^{(\log n)^{O(1)}})$. Let $x \in O(1/(\log n)^{\gamma})$. It follows that there is no correct polynomial-time approximation algorithm for maximum clique that in case the input graph has a clique of size $> n^{1-x}$ would return a clique of size $\Omega(n^x)$.

Suppose that there is a polynomial-time $O(n^{1/2-3x/2})$-approximation algorithm for maximum homeomorphic clique. Let G be the input graph on n vertices. Suppose that G contains a clique of size at least n^{1-x}. Then, the aforementioned algorithm would find a homeomorphic clique $\bar{H}$ in G having $\Omega(n^{1/2+x/2})$ clique vertices. It follows by Lemma 3 that one could determine a clique of size $\Omega(n^x)$ in $\bar{H}$, in polynomial time. We obtain a contradiction.

Similarly, our upper bound on polynomial-time approximability of maximum homeomorphic clique follows from that best for maximum clique [10] by Lemma 3.

Theorem 3. *Maximum homeomorphic clique can be approximated within* $O(n \log\log n/\log^{3/2} n)$ *in polynomial time.*

Proof. The aforementioned best known polynomial-time approximation algorithm for maximum clique achieves the ratio $O(n \log^2 \log n/\log^3 n)$ [10]. Let h be the maximum size of homeomorphic clique in the input graph. By Lemma 3, the graph has a clique of size $\Omega(h^2/n)$. By applying the approximation algorithm for maximum clique to the graph, we obtain a clique of size $\Omega(h^2 \log^3 n/(n \log\log n)^2)$. Thus, in particular for $h = \Omega(n/x)$ where $x \le \log^{3/2} n/\log\log n$, we obtain a clique of size $\Omega(\log^3 n/(x \log\log n)^2)$ and consequently the approximation factor $O(nx \log^2 \log n/\log^3 n)$ for maximum homeomorphic clique in this case. The factor achieves the minimum for $x = \log^{3/2} n/\log\log n$. Hence we obtain the theorem.

5 Subgraph homeomorphism for special guest graphs

We begin from noting that we can use a minor embedding of K_q in a graph to construct a topological embedding of any subgraph of K_q having vertex degrees not exceeding three in the graph.

Theorem 4. *Given a graph* G*, its minor isomorphic to* K_q *and a subgraph* H *of* K_q *whose maximum degree is at most three, one can find a topological embedding of* G *in* H *in time linear in the size of* G*.*

Proof. Let ϕ be the mapping from the vertices of K_q to the subsets of the vertex set of G and from the edges of K_q to edges of G that defines the K_q-minor of G. For each vertex v of H, find a spanning tree T_v of the subgraph induced by $\phi(v)$. For each edge (v, w) of H, where $(v', w') = \phi(v, w)$, mark v' in T_v and w' in T_w. Next, for each vertex v of H prune T_v to the union U_v of the paths in T_v interconnecting at most three marked vertices. It is clear that U_v has the form of either three simple paths meeting at a joint endpoint or just a simple path. By taking the union of the pruned trees U_v over the vertices of H and the ϕ-images of the edges of H, we obtain a subgraph of G homeomorphic with H.

By combining Lemma 2 with Theorem 4, we obtain the next theorem.

Theorem 5. *Let* $1 \le q \le n$ *and let* H *be a subgraph of* K_q *of maximum degree not exceeding three. There is a polynomial-time algorithm which for any graph* G *on* n *vertices produces either a topological embedding of* H *in* G *or a path decomposition of* G *having width* $O(q\sqrt{n} \log^{1.5} n)$.

Fact 5 immediately yields the following lemma.

Lemma 4. *Given a graph G on n vertices whose treewidth does not exceed l, and a family F of k graphs of maximum degree $O(1)$ and treewidth not exceeding l, each having at most n vertices, one can find a maximum vertex cardinality member of F that can be topologically embedded in G as well as its topological embedding in G in time $O(n^{l+2}k)$.*

By combining Theorem 5 with Lemma 4, we obtain our next main result.

Theorem 6. *Let G be a graph on n vertices, let $1 \le q \le n$, and let F be a sequence of graphs H_i, $i = 1, ..., n$, where H_i has i vertices, maximum degree at most three and treewidth $O(q\sqrt{n}\log^{1.5} n)$. One can produce either a topological embedding of H_q in G in polynomial time or a maximum vertex cardinality member of F that can be topologically embedded in G together with its topological embedding in G in time $2^{O(q\sqrt{n}\log^{2.5} n)}$.*

Note that in particular simple cycles and simple paths, having treewidth 2 and 1, respectively, satisfy the requirements on the members in the sequence F. Hence, we obtain the following spectacular corollary.

Corollary 1. *Let G be a graph on n vertices, and let $1 \le q \le n$. One can produce either a simple cycle in G of length not less than q in polynomial time, thus yielding n/q polynomial-time approximation to the longest cycle problem and $(n-1)/(q-1)$ polynomial-time approximation to the longest path problem, or a longest cycle and a longest path of G in time $2^{O(q\sqrt{n}\log^{2.5} n)}$.*

Final remark

Our result on approximability of the clique minor containment can be easily extended to include non necessarily complete guest graphs which are hard to split.

Acknowledgments

The first author is grateful to anonymous ICALP 2004 referees for valuable comments which were especially helpful in overcoming a flaw in the proof of an earlier version of Theorem 1. The authors are also grateful to Anders Dessmark and Andrzej Proskurowski for sharing their knowledge on subgraph embedding algorithms for bounded treewidth graphs, to Eva-Marta Lundell for pointing the minor separator theorem, and to Andreas Björklund and Thore Husfeldt for raising their interest in the longest path and cycle problems.

References

1. N. Alon, P. Seymour, and R. Thomas. A separator theorem for graphs with an excluded minor and its applications. Proc. 22th Ann. Symposium on Theory of Computing, pp. 293-299, ACM, 1990.
2. S. Arnborg. Efficient algorithms for combinatorial problems on graphs with bounded decomposability — A survey. BIT, 25(1), pp. 2-23, 1985.
3. A. Björklund and T. Husfeldt. Finding a Path of Superlogarithmic Length. SIAM Journal on Computing, vol. 32 (2003), no. 6, pp. 1395-1402 (for a preliminary version see proc. ICALP'2002, pp. 985-992).
4. H. Bodlaender. A partial k-arboretum of graphs with bounded treewidth. Theoretical Computer Science, 209 (1998), pp. 1-45.
5. H. Bodlaender and R.H. Möhring. The pathwidth and treewidth of cographs. SIAM J. Disc. Meth., 6 (1993), pp. 181-188.
6. R.G. Downey and M.R. Fellows. Parametrized Complexity, Monographs in Computer Science, Springer 1997.
7. L. Engebretsen and J. Holmerin. Clique is hard to approximate within $n^{1-o(1)}$. Proc. 27th ICALP, pp. 2-12, 2000.
8. D. Eppstein. The traveling salesman problem for cubic graphs. Proc. 8th WADS, LNCS 2748, Springer, pp. 307-318, 2003.
9. T. Feder, R. Motwani, and C.S. Subi. Approximating the longest cycle problem in sparse graphs. SIAM Journal on Computing, 31(5), pp. 1596-1607, 2002.
10. U. Feige. Approximating maximum clique by removing subgraphs. Manuscript, March 10, 2002.
11. H. Gabow and S. Nie. Finding a long directed cycle. Proc. 15th SODA, 2004.
12. M.R. Garey and D.S. Johnson. Computers and Intractability. A Guide to the Theory of NP-completeness. W.H. Freeman and Company, New York 2003.
13. A. Gupta and N. Nishimura. Sequential and Parallel Algorithms for Embedding Problems on Classes of Partial k-Trees. Proc. SWAT 1994, LNCS, Springer, pp. 172-182.
14. A. Gupta and N. Nishimura. The complexity of subgraph isomorphism for classes of partial k-trees. Theoretical Computer Science, 164, pp. 287-298, 1996.
15. J. Håstad. Clique is hard to approximate within $n^{1-\epsilon}$. Acta Mathematica, 182(1), pp. 105-142, 1999.
16. D. Hochbaum (Eds.). Approximation Algorithms for NP-hard Problems. International Thompson Publishers, ISBN 0-534-94968-1, 1995.
17. R. Impagliazzo, R. Paturi and F. Zane. Which Problems Have Strongly Exponential Complexity? Proceedings 1998 Annual IEEE Symposium on Foundations of Computer Science, pp 653-663, 1998.
18. D. Karger, R. Motwani, and G.D.S. Ramkumar. On approximating the longest path in a graph. Algorithmica, 18(1), pp. 82-98, 1997.
19. S. Khot. Improved inapproximability results for MaxClique, chromatic number, and approximate graph coloring. Proc. 42nd FOCS, pp. 600-609, 2001.
20. J. Matousek and R. Thomas. On the complexity of finding iso- and other morphisms for partial k-trees. Discrete Mathematics, 108, pp. 343-364, 1992.
21. S. Plotkin, S. Rao, and W.D. Smith. Shallow excluded minors and improved graph decomposition. 5th Symposium on Discrete Algorithms, pp. 462-470, 1994.
22. N. Robertson and P.D Seymour. Graph minors. II. Algorithmic aspects of tree-width. Journal of Algorithms, 7(3), pp. 309-322, 1986.
23. N. Robertson and P.D. Seymour. Graph minors. XIII. The disjoint paths problem. J. Combinatorial Theory, Ser. B., Vol. 63 (1995) pp. 65-110.

ECCC ISSN 1433-8092
http://www.eccc.uni-trier.de/eccc
ftp://ftp.eccc.uni-trier.de/pub/eccc
ftpmail@ftp.eccc.uni-trier.de, subject 'help eccc'