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Single level conjecture fails also for unbounded fanin circuits

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Theorem. Let H be a bipartite $n \times n$ Sylvester graph, and G be its saturated extension. Then a minimal monotone depth-3 circuit representing H (resp. computing f_G) is by a factor of $\Omega((\log n)^{1/2-o(1)})$ larger than a minimal general monotone circuits representing H (resp. computing f_G). For formulas (fanout-1 circuits) both gaps are at least $\Omega((\log n)^{2-o(1)})$.

Proof. The characteristic function of a Sylvester $n \times n$ graph with $n = 2^m$ is the inner product function $IP_m(x,y) = \bigoplus_{i=1}^m x_i y_i$. This function has a (trivial) non-monotone circuit with fanin-2 gates over the basis $\{\vee, \wedge, \neg\}$ of linear (in m) size. By the Magnification Lemma from [1], the graph H can be represented by a monotone unbounded fanin circuit of size $O(m) = O(\log n)$.

On the other hand, Lokam in [3] has proved that every monotone depth-3 *formula* representing an $n \times n$ Hadamard graph (and hence, also for any $n \times n$ Sylvester graph) has at least $\Omega\left((\log n)^3/(\log\log n)^5\right)$ gates. For depth-3 *circuits*, this implies a lower bound $\Omega((\log n)^{3/2-o(1)})$: just take the maximum of the fanins of gates on the top and middle level.

This implies the gap $\Omega((\log n)^{1/2-o(1)})$ for graphs. The same gap for the quadratic function f_G follows from Lemma 3.8 from [2], according to which, f_G can also be computed by a monotone unbounded fanin circuit of size $O(\log n)$. □

This also gives a partial answer to a question of Pudlák, Rödl and Savický [4] (cf. also Problem 8.3 of [2]).

References

[1] S. Jukna, On graph complexity, ECCC Reprot Nr. 5 (2004). To appear in: Combin. Probab. Comput.

[2] S. Jukna, Disproving the single level conjecture, ECCC Report Nr. 21, Revision 2 (2005). To appear in: SIAM J. Comput.

[3] Lokam, S. V. Graph complexity and slice functions, Theory of Comput. Syst., 36(1) (2003) 1–88.

[4] P. Pudlák, V. Rödl, P. Savický, Graph complexity, Acta Inform., 25 (1988) 515–535.

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