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Generic yet Practical (Statistical) Zero-Knowledge from any Public-Coin HVZK[*][†]

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Abstract

In this work, we present a generic yet practical transformation from any public-coin honest-verifier zero-knowledge (HVZK) protocols to normal zero-knowledge (ZK) arguments. By "generic", we mean that the transformation is applicable to any public-coin HVZK protocol under any one-way function (OWF) admitting Σ-protocols. By "practical" we mean that the transformation does not go through general $\mathcal{NP}$-reductions and only incurs minimal additional rounds. In particular, if the starting public-coin HVZK protocols and the underlying Σ-protocols are practical, the transformed ZK arguments are also practical. In addition, our transformation also preserves statistical/perfect zero-knowledge.

To this end, we develop generic yet practical three-round perfectly-hiding *equivocal* (string) commitment scheme under *any* OWF admitting Σ-protocols, which is of independent value. To our knowledge, it is also the first constant-round statistically-hiding (not necessarily to be trapdoor or equivocal) commitment scheme based on any OWF admitting Σ-protocols. We show that the round-complexity (i.e., three rounds) is optimal for black-box equivocal commitment schemes. Along the way, we also make some clarifications that seem to be possibly of independent interest. In particular, we identify and clarify a seemingly important implication (to the elegant interaction between cryptography and complexity theory): Σ-protocols may witness non-$\mathcal{NP}$-Completeness.

1 Introduction

Zero-knowledge (ZK) protocols are remarkable since they allow a prover to validate theorems to a verifier without giving away any other knowledge (i.e., computational advantage). This notion was suggested by Goldwasser, Micali and Rackoff [42] and its generality was demonstrated by Goldreich, Micali and Wigderson [39]. Since its introduction ZK has found numerous and extremely useful applications, and by now it has been playing a central role in modern cryptography.

ZK protocols for general languages (i.e., $\mathcal{NP}$) constitute an important plausibility result since many important statements are in $\mathcal{NP}$. But, they are normally hard to be directly employed in practice, particularly due to the underlying general $\mathcal{NP}$-reductions. In addition, ZK has many direct efficient applications (mainly employing number-theoretic statements). In particular, a very large number of protocols, named public-coin honest verifier zero-knowledge HVZK protocols, are developed directly for specific number-theoretic languages, which preserves the ZK property only with respect to *honest* verifiers (i.e., they are not normal ZK) but are highly practical. Thus, it's naturally desirable to develop a *generic yet practical* transformation from *any* public-coin HVZK protocols to (normal) ZK protocols.

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[†] Preliminary version of this work appeared in ECCC Report No. 2005-162 (that can be traced back to the incomplete work of [65]). The generic yet practical equivocal commitment scheme based on any OWF admitting Σ-protocols was ever presented in a submission to ICALP05 (Track C chaired by Yung). The details of result strengthening and extension made in the current version, in comparison with the original ECCC version, are presented in Appendix B.

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1.1 Our contributions

In this work, we present a generic yet practical transformation from any public-coin honest-verifier zero-knowledge (HVZK) protocols to normal zero-knowledge (ZK) arguments. By "generic", we mean that the transformation is applicable to any public-coin HVZK protocol under any one-way function (OWF) admitting Σ-protocols. (*Note that the set of OWFs admitting Σ-protocols is large, which in particular includes both DLP and RSA.*) By "practical" we mean that the transformation does not go through general $\mathcal{NP}$-reductions and only incurs additionally one round (for public-coin HVZK protocols of odd number of rounds) or two rounds (for public-coin HVZK protocols of even number of rounds). In particular, if the starting public-coin HVZK protocols and the underlying Σ-protocols are practical, the transformed ZK arguments are also practical. We remark that the additional round-complexity incurred by our transformation is *minimal* in the sense that: There are a very large number of three-round public-coin HVZK protocols (e.g., Σ-protocols) and 2-round public-coin HVZK protocols (as we shall see, any language admits non-interactive zero-knowledge protocols in the common random string model also admits 2-round public-coin HVZK protocols); But, as shown in [36], only languages in $\mathcal{BPP}$ have three-round zero-knowledge protocols.

To this end, we develop generic yet practical three-round perfectly-hiding *equivocal* (string) commitment scheme under *any* OWF admitting Σ-protocols, which is of independent value. To the best of our knowledge, it is also the *first* constant-round statistically-hiding (not necessarily to be trapdoor or equivocal) commitment scheme based on *any* OWF admitting Σ-protocols. The construction is *conceptually simple*, and its DLP or RSA based instantiations need only about 8 exponentiations by each participant for both commitment and decommitment. We show in this work that the round-complexity (i.e., three rounds) is optimal for black-box equivocal commitment schemes.

We also make clarifications on the relationship between trapdoor commitments and equivocal commitments, the necessity of preimage-verifiable OWF in Damgård's paradigm of achieving commitments with Σ-protocols. In particular, we identify and clarify a seemingly important implication (to the elegant interaction between cryptography and complexity theory): Σ-protocols may witness non-$\mathcal{NP}$-Completeness; in other words, a Σ-protocol (without intractability assumption or only with general complexity assumption) may just be a proof for showing that the language is not $\mathcal{NP}$-complete (detailed discussions are presented in Section 3.5). These clarifications might also be of independent interest.

1.2 Related works and comparisons

Converting HVZK protocols into normal ZK protocols has attracted a series of extensive research efforts (and the idea of reducing the problem of security for arbitrary parties to the case of honest parties can be traced back to the works in secure distributed computing [63, 38]). Converting any (whether public-coin or not) HVZK protocols into normal ZK protocols was first studied by Bellare, Micali and Ostrovsky [3] under the DLP assumption, and the intractability assumption was further reduced to any one-way permutation by Ostrovsky, Venkatesan and Yung [58]. The BMO and OVY transformations preserve perfect/statistical ZK. The works of Damgård, Goldreich, Okamoto and Wigderson [19, 57] further achieved that, under any OWF, any (whether public-coin or not) HVZK protocols can be transformed into *public-coin* normal ZK protocols. The transformation of [19, 57] preserves statistical ZK, but not perfect ZK. Damgård first presented a transformation from any *constant-round* public-coin HVZK protocols to normal ZK protocols *without intractability assumptions* [17], by using the interactive hashing technique employed in [58]. The transformation of [17] preserves perfect/statistical ZK, and also proof/argument of knowledge. This (unconditional) approach was carried to its climax in [40] by showing that any public-coin HVZK *proofs* can be transformed into normal *public-coin* ZK proofs *without intractability assumptions*, by a deep investigation of the Random Selection technique employed in [19]. Combined with the result of [57] (i.e., any language that admits honest-verifier statistical ZK proofs also admits public-coin honest-verifier statistical ZK proofs), it established that the class of languages admitting honest-verifier statistical ZK proofs coincides with the class of languages admitting statistical ZK proofs in general. The transformation of [40] preserves perfect/statistical, but it is not provably applicable to HVZK arguments. These (earlier) works show important *general* methodology of achieving ZK protocols in general from HVZK protocols, and demonstrate elegant intersections between cryptography and complexity theory. But these transformations may still be not efficient enough for practice, partially due to the high (additional) round-complexity incurred. In more details, the BMO transformation [3] incurs $c \cdot m(n)$ additional rounds, where c is a constant and $m(n)$ denotes the round-complexity of the starting HVZK protocol on common inputs of length n (which implies the number of incurred rounds is non-constant for non-constant-round starting HVZK protocols); All other transformations presented in [58, 17, 19, 57, 40] incur at least *non-constant* additional rounds in getting normal ZK protocols of negligible soundness error (actually, they incur polynomially many additional rounds for achieving normal ZK protocols of exponentially vanishing soundness error). Other inefficiencies are due to the underlying general $\mathcal{NP}$-reductions (e.g., in the transformations of [3, 58]) and/or the (inefficient) general cryptographic primitives (e.g., OWF-based commitments, et al). In comparison, our transformation is a *practically implementable* one (under any OWF admitting Σ-protocols), transforming any public-coin HVZK protocols into normal ZK arguments with minimal additional rounds incurred and without going through general $\mathcal{NP}$-reductions. Our transformation preserves perfect/statistical ZK. In addition, our transformation also preserves argument of knowledge for Σ-protocols (that are a special case of public-coin HVZK). Note that for HVZK protocols used in practice, the most often case is the *public-coin* ones without $\mathcal{NP}$-reductions (mainly for number-theoretic languages).

Recently, Micciancio and Petrank presented a *practically implementable* transformation from any public-coin HVZK protocols to normal ZK protocols without general $\mathcal{NP}$-reductions under the decisional Diffie-Hellman (DDH) assumption [52]. Note that our transformation converts any public-coin HVZK protocols into normal ZK arguments under any OWF admitting Σ-protocol (that includes, in particular, both DLP and RSA). For the additional round-complexity incurred, the Micciancio-Petrank transformation incurs three rounds (for public-coin HVZK protocols of odd number of rounds) or four rounds (for public-coin HVZK protocols of even number of rounds)*. In comparison, our transformation only incurs minimal additional rounds (specifically, one round for public-coin HVZK protocols of odd number of rounds or two rounds for public-coin HVZK protocols of even number of rounds). For additional computational complexity incurred, the Micciancio-Petrank transformation (that is based on DDH) incurs additional 2 modular exponentiation operations for each round of the original public-coin HVZK protocol. The DLP (that is weaker than DDH) or RSA based implementations of our transformation incur the same additional computational complexity (i.e., 2 exponentiations for each round of the original protocol). The Micciancio-Petrank transformation can be extended to render concurrent ZK protocols by directly applying the Richardson-Kilian paradigm [60], with non-constant additional rounds incurred as constant-round black-box concurrent ZK protocols exists only for trivial languages [11]. This property does not hold for our transformation. In turn, our transformation preserves perfect/statistical ZK, a property does not hold for the Micciancio-Petrank transformation. The Micciancio-Petrank transformation only preserves computational ZK. As a critical tool, [52] achieved a DDH-based perfectly-binding simulatable commitment scheme. In this work, we achieve perfectly-hiding three-round (that is optimal) equivocal commitment schemes under any OWF admitting Σ-protocols. We also note that the soundness of the transformed ZK protocols by the Micciancio-Petrank transformation is trivially guaranteed by the nicely explored DDH-based perfectly-binding (simulatable) commitment scheme, while the soundness proof of the ZK protocols by our transformation turns out to be (a bit) complicated and subtle.

The notion of equivocal commitments is proposed in [2], where the construction of such a scheme was left over there as an open problem. Equivocal commitments are firstly achieved in [23] by a modification of Naor's scheme [53] in the common reference string model. Equivocal commitments in the standard model without setup assumptions are firstly achieved in [25]. The approach of [25] is to nicely modify the schemes of [53, 7] with a coin-tossing on the top, and thus the resulting equivocal commitment schemes run in at least four rounds. Very recently, Katz and Ostrovsky nicely present a zero-knowledge based solution for equivocal commitments [50], which can be easily modified to run in three rounds. The Katz-Ostrovsky scheme is a bit commitment scheme and to commit a string the scheme will be executed

*In [52], the authors only analyzed the additional round complexity incurred for HVZK of odd number of rounds.

in parallel, and thus may not be efficient for committing strings. Note that our scheme is equivocal string commitment scheme, and can be practically instantiated with a very small constant number of exponentiations. Also, although three-round equivocal commitment scheme is (implicitly) known in [50], a proof of the lower-bound of round-complexity (i.e., three rounds are optimal for equivocal commitments) is unknown previously, to the best of our knowledge.

Recent celebrated advances of general complexity assumption based statistical ZK and commitments. Recently, celebrated advances are progressed on statistical ZK and statistically-hiding commitments from general complexity assumptions, in particular from any OWF [45, 56, 46]. Known general assumption based statistical ZK and commitments are feasible inefficient constructions and are of non-constant rounds [54, 45, 56, 46]. Furthermore, it is very recently proved that non-constant round-complexity is intrinsic to black-box constructions of statistical commitments from any one-way permutations (and even from trapdoor permutations) [44] (which improves the previous round-complexity lower-bound of general complexity assumption based statistical commitments [30, 62]).

2 Preliminaries

We use standard notations and conventions below for writing probabilistic algorithms, experiments and interactive protocols. If A is a probabilistic algorithm, then $A(x_1, x_2, \cdots; r)$ is the result of running A on inputs $x_1, x_2, \cdots$ and coins r. We let $y \leftarrow A(x_1, x_2, \cdots)$ denote the experiment of picking r at random and letting y be $A(x_1, x_2, \cdots; r)$. If S is a finite set then $x \leftarrow S$ is the operation of picking an element uniformly from S. (If S is a finite set we also denote by $|S|$ the size of S.) If α is neither an algorithm nor a set then $x \leftarrow \alpha$ is a simple assignment statement. By $[R_1; \cdots; R_n : v]$ we denote the set of values of v that a random variable can assume, due to the distribution determined by the sequence of random processes $R_1, R_2, \cdots, R_n$. By $\Pr[R_1; \cdots; R_n : E]$ we denote the probability of event E, after the ordered execution of random processes $R_1, \cdots, R_n$.

Let $\langle P, V \rangle$ be a probabilistic interactive protocol, then the notation $(y_1, y_2) \leftarrow \langle P(x_1), V(x_2) \rangle(x)$ denotes the random process of running interactive protocol $\langle P, V \rangle$ on common input x, where P has private input x_1, V has private input x_2, y_1 is P's output and y_2 is V's output. We assume, w.l.o.g., that the output of both parties P and V at the end of an execution of the protocol $\langle P, V \rangle$ contains a transcript of the communication exchanged between P and V during such execution.

Definition 2.1 (one-way function) *A function $f : \{0,1\}^* \longrightarrow \{0,1\}^*$ is called a one-way function (OWF) if the following conditions hold:*

1. *Easy to compute: There exists a (deterministic) polynomial-time algorithm A such that on input x algorithm A outputs $f(x)$ (i.e., $A(x) = f(x)$).*

2. *Hard to invert: For every probabilistic polynomial-time PPT algorithm A', every positive polynomial $p(\cdot)$, and all sufficiently large n's, it holds $\Pr[A'(f(U_n), 1^n) \in f^{-1}(f(U_n))] < \frac{1}{p(n)}$, where U_n denotes a random variable uniformly distributed over $\{0,1\}^n$.*

A OWF f is called *preimage-verifiable* if there exists a polynomial-time computable predicate, $D_f : \{0,1\}^* \longrightarrow \{0,1\}$, such that for any string y, $D_f(y) = 1$ if and only if there exists an x such that $y = f(x)$.

Discussions on preimage-verifiable OWF. Below, with RSA as an example, we give more clarifications about the relationship among normal OWF, preimage-verifiable OWF and 1-1 OWF. (More detailed clarifications can be found in [37, 64].) The difference between normal OWF and length-preserving 1-1 OWF lies in the domain of the function. Given a normal OWF from domain D to Range R, the 1-1 property of the function is defined however with respect to $\{0,1\}^* \longrightarrow \{0,1\}^*$. In other words, length-preserving 1-1 OWF is a single object, while a normal OWF could be a member of a function family. This in particular implies that any length-preserving 1-1 OWF is trivially a preimage-verifiable OWF.

For example, given a function description (N, e) where $N = pq$ for two distinct primes p and q and $gcd(e, \phi(N)) = 1$, define this generic RSA-based function (denoted by f_{GRSA}) to be $f_{GRSA}(x) = x^e$ mod N. Then, this function f_{GRSA} (specified by (N, e)) is a normal OWF (actually OWP) from Z_N^* to Z_N^*. But, this function is NOT a preimage-verifiable OWF, because given a specific (N, e), one cannot efficiently verify whether $gcd(e, \phi(N)) = 1$ or not.

Now, consider the following *restricted* RSA-based function f_{RRSA} (specified by (N, e) such that N is a composite number and $e > N$ is a prime number). In this case, this function f_{RRSA} is a preimage-verifiable OWF from Z_N^* to Z_N^*, as the additional requirement $e > N$ ensures that $gcd(e, \phi(N)) = 1$. But, this function is clearly not a 1-1 function from $\{0, 1\}^* \longrightarrow \{0, 1\}^*$.

Definition 2.2 (interactive argument system) *A pair of probabilistic polynomial-time interactive machines, $\langle P, V\rangle$, is called an interactive argument system for a language L if the following conditions hold:*

- *Completeness. For every $x \in L$, there exists a string w such that for every string z,* $\Pr[\langle P(w), V(z)\rangle(x) = 1] = 1$.
- *Soundness. For every polynomial-time interactive machine P^*, and for all sufficiently large n's and every $x \notin L$ of length n and every w and z,* $\Pr[\langle P^*(w), V(z)\rangle(x) = 1]$ *is negligible in n.*

An interactive system is called a public-coin system if at each round the prescribed verifier only tosses a predetermined number of coins and send the outcome (random challenge) to the prover.

Definition 2.3 (witness indistinguishability WI [29]) *Let $\langle P, V\rangle$ be an interactive system for a language $L \in \mathcal{NP}$, and let R_L be the fixed $\mathcal{NP}$ witness relation for L. That is, $x \in L$ if there exists a w such that $(x, w) \in R_L$. We denote by $view_{V^*(z)}^{P(w)}(x)$ a random variable describing the contents of the random tape of V^* and the messages V^* receives from P during an execution of the protocol on common input x, when P has auxiliary input w and V^* has auxiliary input z. We say that $\langle P, V\rangle$ is witness indistinguishable for R_L if for every PPT interactive machine V^*, and every two sequences $W^1 = \{w_x^1\}_{x\in L}$ and $W^2 = \{w_x^2\}_{x\in L}$ for sufficiently long x, so that $(x, w_x^1) \in R_L$ and $(x, w_x^2) \in R_L$, the following two ensembles are computationally indistinguishable by any non-uniform PPT algorithm: $\{x, view_{V^*(z)}^{P(w_x^1)}(x)\}_{x\in L, z\in\{0,1\}^*}$ and $\{x, view_{V^*(z)}^{P(w_x^2)}(x)\}_{x\in L, z\in\{0,1\}^*}$. Namely, for every PPT non-uniform distinguishing algorithm D, every polynomial $p(\cdot)$, all sufficiently long $x \in L$, and all $z \in \{0, 1\}^*$, it holds that*

$$|\Pr[D(x, z, view_{V^*(z)}^{P(w_x^1)}(x) = 1] - \Pr[D(x, z, view_{V^*(z)}^{P(w_x^2)}(x) = 1]| < \frac{1}{p(|x|)}$$

Definition 2.4 (zero-knowledge ZK [42, 33]) *Let $\langle P, V\rangle$ be an interactive system for a language $L \in \mathcal{NP}$, and let R_L be the fixed $\mathcal{NP}$ witness relation for L. That is, $x \in L$ if there exists a w such that $(x, w) \in R_L$. We denote by $view_{V^*(z)}^{P(w)}(x)$ a random variable describing the contents of the random tape of V^* and the messages V^* receives from P during an execution of the protocol on common input x, when P has auxiliary input w and V^* has auxiliary input z. Then we say that $\langle P, V\rangle$ is zero-knowledge if for every probabilistic polynomial-time interactive machine V^* there exists a probabilistic (expected) polynomial-time oracle machine S, such that for all sufficiently long $x \in L$ the ensembles $\{view_{V^*}^{P(w)}(x)\}_{x\in L}$ and $\{S^{V^*}(x)\}_{x\in L}$ are computationally indistinguishable. Machine S is called a ZK simulator for $\langle P, V\rangle$. The protocol is called statistical ZK if the above two ensembles are statistically close (i.e., the variation distance is eventually smaller than $\frac{1}{p(|x|)}$ for any positive polynomial p). The protocol is called perfect ZK if the above two ensembles are actually identical (i.e., except for negligible probabilities, the two ensembles are equal).*

Definition 2.5 (public-coin HVZK) *Let $\langle P, V\rangle$ be a public-coin interactive protocol for a language L in which the prescribed honest verifier V is supposed to send $t-1$, $t \geqslant 2$, random challenges. Denote by α_i, $1 \le i \le t$, the i-th message of honest prover and by c_i, $1 \le i \le t-1$, the i-th random challenge of* *the honest verifier. The honest prover P is a PPT interactive machine that on the common input x, an auxiliary input w and a partial transcript T where T is either an empty string or a sequence of messages $(\alpha_1, c_1, \cdots, c_i)$, $1 \leq i \leq t-1$, outputs the next message α_{i+1}. We denote by $view_V^{P(w)}(x)$ a random variable describing the contents of the random tape of (the honest) V and the messages V receives from P during an execution of the protocol on common input x when P has auxiliary input w. Such a public-coin protocol is called honest verifier zero-knowledge (HVZK) if there exists a probabilistic polynomial time simulator S such that for all sufficiently long $x \in L$ the following ensembles are computationally indistinguishable: $\{S(x)\}_{x \in L}$ and $\{view_V^{P(w)}(x)\}_{x \in L}$.*

In above definition, we have assumed that the prover initiates the protocol. It also can be easily extended to deal with the case that the verifier initiates the protocol.

Definition 2.6 (Σ-protocol [13]) *A three-round public-coin protocol $\langle P, V\rangle$ is said to be a Σ-protocol for a relation R if the following hold:*

- *Completeness. If P, V follow the protocol, the verifier always accepts.*

- *Special soundness. From any common input x of length n and any pair of accepting conversations on input x, (a, e, z) and (a, e', z') where $e \neq e'$, one can efficiently compute w such that $(x, w) \in R$. Here a, e, z stand for the first, the second and the third message respectively and e is assumed to be a string of length k (that is polynomially related to n) selected uniformly at random in $\{0,1\}^k$.*

- *Perfect SHVZK (Special honest verifier zero-knowledge). There exists a probabilistic polynomial-time (PPT) simulator S, which on input x (where there exists a w such that $(x, w) \in R$) and a random challenge string $\hat{e}$, outputs an accepting conversation of the form $(\hat{a}, \hat{e}, \hat{z})$, with the same probability distribution as the real conversation (a, e, z) between the honest $P(w)$, V on input x.*

Σ-protocols have been proved to be a very powerful cryptographic tool and are widely used in numerous important cryptographic applications. A large number of Σ-protocols have been developed in the literature and we recall the Σ-protocol examples for DLP and RSA below.

Σ-protocols vs public-coin HVZK: Although Σ-protocols are a very powerful cryptographic tool, by definition they are actually a very special case of public-coin HVZK (specifically, three-round public-coin special HVZK with special soundness). There are many protocols that are public-coin HVZK but not Σ-protocols. In particular, any non-interactive zero-knowledge NIZK protocol in the common random string model can be trivially modified into a 2-round public-coin HVZK protocol. The idea is simple: we let the (honest) verifier send a random string in the first-round, and then the prover generates the NIZK proof/argument with the first-round message serving as the underlying common random string. It is also known that proving a number is the product of two safe primes has a public-coin HVZK (with round-complexity much larger than 3, and thus not Σ-protocols) [8].

Σ-Protocol for DLP [61]. The following is a Σ-protocol $\langle P, V\rangle$ proposed by Schnorr [61] for proving the knowledge of discrete logarithm, w, for a common input of the form (p, q, g, h) such that $h = g^w \ mod \ p$, where on a security parameter n, p is a uniformly selected n-bit prime such that $q = (p-1)/2$ is also a prime, g is an element in $\mathbf{Z}_p^*$ of order q. It is also actually the first efficient Σ-protocol proposed in the literature.

- P chooses r at random in $\mathbf{Z}_q$ and sends $a = g^r \ mod \ p$ to V.

- V chooses a challenge e at random in $\mathbf{Z}_{2^k}$ and sends it to P. Here, k is fixed such that $2^k < q$.

- P sends $z = r + ew \ mod \ q$ to V, who checks that $g^z = ah^e \ mod \ p$, that p, q are prime and that g, h have order q, and accepts iff this is the case.

Σ-Protocol for RSA [43]. Let n be an RSA modulus and q be a prime. Assume we are given some element $y \in \mathbf{Z}_n^*$, and P knows an element w such that $w^q = y \mod n$. The following protocol is a Σ-protocol for proving the knowledge of q-th roots modulo n.

- P chooses r at random in $\mathbf{Z}_n^*$ and sends $a = r^q \mod n$ to V.
- V chooses a challenge e at random in $\mathbf{Z}_{2^k}$ and sends it to P. Here, k is fixed such that $2^k < q$.
- P sends $z = rw^e \mod n$ to V, who checks that $z^q = ay^e \mod n$, that q is a prime, that $gcd(a, n) = gcd(y, n) = 1$, and accepts iff this is the case.

The OR-proof of Σ-protocols [15]. One basic construction with Σ-protocols allows a prover to show that given two inputs x_0, x_1, it knows a w such that either $(x_0, w) \in R_0$ or $(x_1, w) \in R_1$, but without revealing which is the case. Specifically, given two Σ-protocols $\langle P_b, V_b \rangle$ for R_b, $b \in \{0, 1\}$, with random challenges of, without loss of generality, the same length k, consider the following protocol $\langle P, V \rangle$, which we call Σ_{OR}. The common input of $\langle P, V \rangle$ is (x_0, x_1) and P has a private input w such that $(x_b, w) \in R_b$.

- P computes the first message a_b in $\langle P_b, V_b \rangle$, using x_b, w as private inputs. P chooses e_{1-b} at random, runs the SHVZK simulator of $\langle P_{1-b}, V_{1-b} \rangle$ on input (x_{1-b}, e_{1-b}), and lets $(a_{1-b}, e_{1-b}, z_{1-b})$ be the output. P finally sends a_0, a_1 to V.
- V chooses a random k-bit string s and sends it to P.
- P sets $e_b = s \oplus e_{1-b}$ and computes the answer z_b to challenge e_b using (x_b, a_b, e_b, w) as input. He sends (e_0, z_0, e_1, z_1) to V.
- V checks that $s = e_0 \oplus e_1$ and that conversations (a_0, e_0, z_o), (a_1, e_1, z_1) are accepting conversations with respect to inputs x_0, x_1, respectively.

Theorem 2.1 [15] *The protocol Σ_{OR} above is a Σ-protocol for R_{OR}, where $R_{OR} = \{((x_0, x_1), w) | (x_0, w) \in R_0$ or $(x_1, w) \in R_1\}$. Moreover, for any malicious verifier V^*, the probability distribution of conversations between P and V^*, where w is such that $(x_b, w) \in R_b$, is independent of b. That is, Σ_{OR} is perfectly witness indistinguishable.*

The perfect SHVZK simulator of Σ_{OR} [15]. For a Σ_{OR}-protocol of above form, denote by S_{OR} the perfect SHVZK simulator of it and denote by S_b the perfect SHVZK simulator of the protocol $\langle P_b, V_b \rangle$ for $b \in \{0, 1\}$. Then on common input (x_0, x_1) and a random string $\hat{e}$ of length k, $S_{OR}((x_0, x_1), \hat{e})$ works as follows: It firstly chooses a random k-bit string $\hat{e}_0$, computes $\hat{e}_1 = \hat{e} \oplus \hat{e}_0$, then S_{OR} runs $S_b(x_b, \hat{e}_b)$ to get a simulated transcript $(\hat{a}_b, \hat{e}_b, \hat{z}_b)$ for $b \in \{0, 1\}$, finally S_{OR} outputs $((\hat{a}_0, \hat{a}_1), \hat{e}, (\hat{e}_0, \hat{z}_0, \hat{e}_1, \hat{z}_1))$.

Definition 2.7 (trapdoor (string) commitment scheme TC) *A (normal) trapdoor commitment scheme (TC) is a quintuple of probabilistic polynomial-time (PPT) algorithms TCGen, TCCom, TCVer, TCKeyVer and TCFake, such that*

- *Completeness. $\forall n$, $\forall v$ of length k (where $k = k(n)$ for some polynomial $k(\cdot)$),*
  $\Pr[(TCPK, TCSK) \leftarrow \mathrm{TCGen}(1^n); (c, d) \leftarrow \mathrm{TCCom}(1^n, 1^k, TCPK, v) :$
  $\mathrm{TCKeyVer}(1^n, TCPK) = \mathrm{TCVer}(1^n, 1^k, TCPK, c, v, d) = 1] = 1$.
- *Computational Binding. For all sufficiently large n's and for any PPT adversary A, the following probability is negligible in n (where $k = k(n)$ for some polynomial $k(\cdot)$):*
  $\Pr[(TCPK, TCSK) \leftarrow \mathrm{TCGen}(1^n); (c, v_1, v_2, d_1, d_2) \leftarrow \mathrm{A}(1^n, 1^k, TCPK) :$
  $\mathrm{TCVer}(1^n, 1^k, TCPK, c, v_1, d_1) = \mathrm{TCVer}(1^n, 1^k, TCPK, c, v_2, d_2) = 1 \bigwedge |v_1| = |v_2| = k \bigwedge v_1 \neq v_2]$.
- *Perfect (or Computational) Hiding. $\forall$ $TCPK$ such that* $\mathrm{TCKeyVer}(TCPK, 1^n) = 1$ *and $\forall$ v_1, v_2 of equal length k, the following two probability distributions are identical (or computationally indistinguishable):*
  $[(c_1, d_1) \leftarrow \mathrm{TCCom}(1^n, 1^k, TCPK, v_1) : c_1]$ *and* $[(c_2, d_2) \leftarrow \mathrm{TCCom}(1^n, 1^k, TCPK, v_2) : c_2]$.

- *Perfect (or Computational) Trapdoorness.* $\forall$ $(TCPK, TCSK) \in \{\mathrm{TCGen}(1^n)\}$, $\exists v_1$, $\forall v_2$ *such that* v_1 *and* v_2 *are of equal length* k, *the following two probability distributions are identical (or computationally indistinguishable):* $[(c_1, d_1) \leftarrow \mathrm{TCCom}(1^n, 1^k, TCPK, v_1); d'_2 \leftarrow \mathrm{TCFake}(1^n, 1^k, TCPK, TCSK, c_1, v_1, d_1, v_2) : (c_1, d'_2)]$ *and* $[(c_2, d_2) \leftarrow \mathrm{TCCom}(1^n, 1^k, TCPK, v_2) : (c_2, d_2)]$.

3 Generic yet Practical Equivocal Commitments from Σ-Protocols

In this section, we present *generic yet practical* three-round perfectly-hiding equivocal (string) commitment schemes under any OWF admitting Σ-protocols. By "generic" we mean our equivocal commitments can be implemented under any one-way function (OWF) that admits Σ-protocols (*note that the set of OWFs admitting Σ-protocols is large, which in particular includes both DLP and RSA*). To the best of our knowledge, it is also the *first* constant-round statistically-hiding (not necessarily to be trapdoor or equivocal) commitment scheme based on *any* OWF admitting Σ-protocols; By "practical" we mean our equivocal commitment schemes do not go through general $\mathcal{NP}$-reductions and if the underlying Σ-protocols are practical then the transformed equivocal commitment protocols are also practical. With DLP or RSA as examples, the DLP-based or RSA-based instantiations need only 8 exponentiations by each participant for both commitment and decommitment. We show that three rounds is the lower-bound of round-complexity for black-box equivocal commitment schemes (the lower-bound holds unconditionally).

Along the way, we also clarify in this section the relationship between trapdoor commitments and equivocal commitments, the necessity of preimage-verifiable OWF in Damgård's Σ-protocol-based trapdoor commitments, and identify and clarify a seemingly important implication (to the elegant interaction between cryptography and complexity theory): Σ-protocols may witness non-$\mathcal{NP}$-Completeness; in other words, a Σ-protocol (without intractability assumption or only with general complexity assumption) may just be a proof for showing the language is not $\mathcal{NP}$-complete (detailed discussions are presented in Section 3.5). These clarifications may also be possibly of independent interest.

3.1 Formalization of equivocal commitments

Informally speaking, a commitment scheme is equivocal if it satisfies the following additional requirement. There exists an efficient algorithm, called the simulator, which outputs a transcript leading to a "fake" commitment such that: (1) the "fake" commitment can be decommitted to both 0 and 1; and (2) the distribution of the simulated transcript is indistinguishable from that of the real view of an even malicious commitment receiver in a real execution of the protocol. Equivocal commitments could be viewed as a stronger notion than trapdoor commitments and are widely used as a key ingredient for achieving various advanced cryptographic protocols, e.g., zero-knowledge [54, 25, 26], secure multi-party computation [12, 50], and more advanced commitment schemes (like non-malleable commitments [23, 31, 24, 20], simulation-sound commitments [51] and universally composable commitments [10, 21, 12]), et al.

Now, we present the formal definition of (black-box) equivocal (string) commitments that is the black-box and full version of the definition presented in [25].

Definition 3.1 *Let* $\langle P, V\rangle$ *be an interactive protocol. We say that* $\langle P, V\rangle$ *is a (black-box)* **equivocal** *commitment scheme if it satisfies the following:*

Perfect (or computational) hiding. *For all sufficiently large* n*'s, any PPT adversary* V^* *and any* s, s' *of equal length* k *(where* $k = k(n)$ *for some polynomial* $k(\cdot)$*), the following two probability distributions are identical (or computationally indistinguishable):* $[(\alpha, \beta) \leftarrow \langle P(s), V^*\rangle(1^n, 1^k) : \beta]$ *and* $[(\alpha', \beta') \leftarrow \langle P(s'), V^*\rangle(1^n, 1^k) : \beta']$.

Computational Binding. *For all sufficiently large* n*'s, and any PPT adversary* P^**, the following probability is negligible in* n*:* $\Pr[(\alpha, \beta) \leftarrow \langle P^*, V\rangle(1^n, 1^k); (t, (t, v)) \leftarrow \langle P^*(\alpha), V(\beta)\rangle(1^n, 1^k); (t', (t', v')) \leftarrow \langle P^*(\alpha), V(\beta)\rangle(1^n, 1^k) : |v| = |v'| = k \bigwedge v \neq v']$.

That is, no PPT adversary P^* can decommit the same transcript of the commitment stage to two different values with non-negligible probabilities.

Simulation indistinguishability and equivocability. *There exists a probabilistic polynomial-time S (who works in two stages: the commitment stage S^c and the decommitment stage S^d) such that for any probabilistic polynomial-time algorithm V^*, it holds:*

1. *Indistinguishability. For any string $s \in \{0,1\}^k$, the distributions $T(S)$ and $T(P)$ are computationally indistinguishable, where*

$$T(S) = [(\alpha, \beta) \leftarrow S^c_{V^*}(1^n, 1^k); (t, (t, s)) \leftarrow S^d_{V^*(\beta)}(1^n, 1^k, \alpha, s) : (\beta, (t, s))]$$

$$T(P) = [(\alpha, \beta) \leftarrow \langle P(s), V^* \rangle(1^n, 1^k); (t, (t, s)) \leftarrow \langle P(\alpha, s), V^*(\beta) \rangle(1^n, 1^k) : (\beta, (t, s))]$$

.

That is, for any string s of length k, S outputs a simulated transcript (of both the commitment stage and the decommitment stage) that is indistinguishable from the view of V^* in real interactions with P when P commits s in the commitment stage and then reveals s in the decommitment stage, where the simulation of the commitment stage (i.e, S^c) is independent of the string s that is only given to S^d after the simulation of S^c is finished.

2. *Equivocability. For all constant c, all sufficiently large n, any string $s \in \{0,1\}^k$, where $k = k(n)$ for some polynomial $k(\cdot)$, it holds that $|p_0 - p_1| \leq n^{-c}$, where p_0, p_1 are, respectively,*

$$\Pr[(\alpha, \beta) \leftarrow S^c_{V^*}(1^n, 1^k); (t, (t, v)) \leftarrow S^d_{V^*(\beta)}(1^n, 1^k, \alpha, s) : v = s],$$

$$\Pr[(\alpha, \beta) \leftarrow \langle P(s), V^* \rangle(1^n, 1^k); (t, (t, v)) \leftarrow \langle P(\alpha, s), V^*(\beta) \rangle(1^n, 1^k) : v = s].$$

That is, S can decommit the simulated transcript of the commitment stage to any *value (of length k) correctly.*

Trapdoor commitments versus equivocal commitments. We first remark that the notions of trapdoor commitments and equivocal commitments are significantly different in nature. In particular, the trapdoorness property of a trapdoor commitment scheme is defined with respect to *honest* commitment receiver. In other words, the trapdoorness property of a trapdoor commitment scheme is defined with respect to *well-formed $TCPK$*. For the Damgård's Σ-protocol based trapdoor commitment scheme (described in Section 2), the $TCPK$ is $y = f(x)$. Then, the trapdoorness property says that for well-formed $y = f(x)$ (which guarantees the existence of the preimage x), if one knows the preimage x then it can equivocate commitments at its wish. But, it does not guarantee that the trapdoorness property still holds if y is maliciously formed. For example, a malicious commitment receiver may send a maliciously formed y' such that there exists no preimages of y', while it is hard for the honest commitment sender to verify whether or not the preimages of y' exist. For example, consider the SQUARE one-way permutation (over the quadratic residues): $f(x) = x^2 \mod N$, where $N = p \cdot q$ and $p = q = 3 \mod 4$. Then the malicious commitment receiver may form y' to be a non-square such that there exists no x satisfying $y' = x^2 \mod N$. Note that the honest commitment sender (that is a PPT algorithm) cannot efficiently verify whether y' is a square or not, and thus the trapdoorness property becomes meaningless in this case. But, for equivocal commitments, the equivocability is defined with respect to *any malicious commitment receiver*, whether honest or dishonest. This is a difference in nature between trapdoor commitments and equivocal commitments.

3.2 Warm-up for equivocal commitments based on *any* OWF admitting Σ-protocols

Before describing our generic yet practical equivocal (string) commitment scheme, we first provide the warm-up for achieving equivocal commitments from Σ-protocols.

Damgård's paradigm for achieving perfectly-hiding trapdoor commitment schemes from any *preimage-verifiable* OWF admitting Σ-protocols [16]. Recall that by the *perfect* SHVZK property, the distribution of $(\hat{a}, \hat{e}, \hat{z})$ outputted by the SHVZK simulator $S(x, \hat{e})$ on a true statement $x \in L$ and a random string $\hat{e}$ of length k is *identical* to that of real interaction transcript (a, e, z) between the honest P, V on x. But, in the real conversation (a, e, z), a is independent of e and thus reveals no information of e. This means that in the simulated transcript $(\hat{a}, \hat{e}, \hat{z})$, $\hat{a}$ reveals also no information of $\hat{e}$ (perfect hiding). Furthermore, given $\hat{a}$, the simulator S also cannot answer a different challenge $\hat{e}' \neq \hat{e}$ because of the special soundness of Σ-protocols (this is just the binding property of commitments). The important observation here is that even if $\hat{e}$ is an *arbitrary* string of length k (rather than a random string), $\hat{a}$ still perfectly hides $\hat{e}$. This is so because of the *perfect* SHVZK property of Σ-protocols. This directly brings us the following perfectly-hiding trapdoor commitment scheme $\langle P, V \rangle$ [16].

Round-1. On a security parameter n, let f be a preimage-verifiable OWF that admits Σ-protocols (with random challenges of length k that is polynomially related to n). Then, the commitment receiver V randomly selects an element x of length n in the domain of f, computes $y = f(x)$, and sends y to the commitment sender P.

Round-2. P first checks that y is well-formed (i.e., whether preimages of y exist) and aborts if the check fails. Otherwise, to commit a message m of length k, P runs the SHVZK simulator $S(y, m)$ to get a simulated transcript, denoted $(\hat{a}, m, \hat{z})$, pretending that it "knows" the preimage of y. P sends $\hat{a}$ to V as the commitment.

Decommitment Stage. P reveals $(m, \hat{z})$ and V accepts if $(\hat{a}, m, \hat{z})$ is an accepting conversation on y.

The perfectly-hiding and computationally-binding properties of the above scheme can be easily checked according to the above arguments. The trapdoorness property is from the observation that: if one knows the preimage of the $f(x)$ (that can be efficiently checked due to the preimage-verifiability of f), then it can equivocate $\hat{a}$ at its wish.

On the critical role of preimage-verifiability of OWF. We note that preimage-verifiable OWF plays an essential role for the perfectly-hiding property of the above commitment scheme. The underlying reason is: for Σ-protocols, the simulated transcript output by the SHVZK simulator may be arbitrarily different from the real interaction transcript w.r.t. false common input, i.e., when y is maliciously generated such that no preimage exists. To make it clearer, consider the commitment scheme got by applying Damgård paradigm on the Σ-protocol for Quadratic Residues [43]: for such commitment scheme, if the malicious receiver sends a non-quadratic residue at the first round, then the commitment to 0 is always a quadratic residue, but the commitment to 1 is always a non-quadratic residue, which is clearly not perfectly-hiding. Similar issue also arises when applying Damgård's paradigm on the Σ-protocol for Graph Isomorphism (GI) [33]: suppose the receiver sends a pair of non-isomorphic graphs (G_0, G_1) at the first-round, then a commitment to a bit b is always a graph isomorphic to G_b[†].

This raises the question that whether constant-round perfectly-hiding trapdoor (actually more advanced, say, equivocal) commitments could be implemented from any (not necessarily preimage-verifiable) OWF admitting Σ-protocols, which is answered in next section.

High-level discussions of our solution. Starting from the Damgård's preimage-verifiable OWF based 2-round trapdoor commitment scheme, a first intuitional attempt for constructing equivocal commitment schemes from any OWF admitting Σ-protocols might be that: we let the verifier not only send $f(x)$ but also prove to the prover the knowledge of the preimage x by executing the Σ-protocol on common input $f(x)$. But, this simple solution does not yield an equivocal commitment scheme. Actually, it even does not constitute a normal commitment scheme. The reason is that we cannot prove the binding property of the resulting protocol, as Σ-protocol is only HVZK (rather than normal ZK). Then, a second attempt is: we let the verifier prove to the prover the knowledge of the preimage by executing a normal ZK proof/argument of knowledge protocol on the top. But, this solution is inefficient: ZK proof/argument of knowledge protocols are normally less efficient and run in at least

[†] We note that the situation with GI-based commitments is a bit different, as we do not know how to well define OWFs from GI.

The Σ_{OR}-based equivocal commitment scheme $\langle P, V\rangle$

Commitment Stage. The commitment stage consists of the following two phases:

Phase-1. On a security parameter n, the commitment receiver V selects a OWF f_V that admits Σ-protocols, randomly selects two elements in the domain of f_V, x_V^0 and x_V^1 of length n, and computes $y_V^0 = f_V(x_V^0)$ and $y_V^1 = f_V(x_V^1)$. Finally, V sends (y_V^0, y_V^1) to the commitment sender P, and proves to P that it knows the preimage of either y_V^0 or y_V^1 by executing the Σ_{OR}-protocol on (y_V^0, y_V^1) and playing the role of the knowledge prover. The witness used by V during the execution of the Σ_{OR}-protocol is x_V^b for a randomly chosen bit b in $\{0,1\}$.

Phase-2. Suppose $m \in \{0,1\}^k$ be the message to be committed, P runs the perfect SHVZK simulator $S_{OR}((y_V^0, y_V^1), m)$ (as described in Section 2) to get a simulated transcript, denoted $((\hat{a}_0, \hat{a}_1), m, (\hat{e}_0, \hat{z}_0, \hat{e}_1, \hat{z}_1))$, pretending that it "knows" either the preimage of y_V^0 or the preimage of y_V^1. Finally, P sends $(\hat{a}_0, \hat{a}_1)$ to V while keeping $(m, (\hat{e}_0, \hat{z}_0, \hat{e}_1, \hat{z}_1))$ in secret as the decommitment information. (We remark that P aborts in case the Σ_{OR}-protocol of Phase-1 is not valid or V does not successfully finish the Σ_{OR}-protocol.)

Decommitment Stage. P reveals $(m, (\hat{e}_0, \hat{z}_0, \hat{e}_1, \hat{z}_1))$. V checks that $m = \hat{e}_0 \oplus \hat{e}_1$ and that for both $b = 0$ and $b = 1$ $(\hat{a}_b, \hat{e}_b, \hat{z}_b)$ is an accepting conversation with respect to y_V^b. V accepts if all the above are valid, otherwise it rejects.

Figure 1. Generic yet practical round-optimal equivocal string commitments
under any OWF admitting Σ-protocols

four rounds (which means the resulting equivocal commitment scheme will also run in at least four rounds).

Our observation here is that: normal ZK proof/argument of knowledge protocols are not necessary for the above approach of achieving equivocal commitment schemes from Σ-protocols. Actually, WI protocols (i.e., Σ_{OR}) could suffice for our purpose with some careful investigations. In particular, to guarantee the binding property and the equivocability property simultaneously, as we shall show, both the verifier and the prover need to employ the OR-technique.

3.3 The actual construction and analysis

Now, we present the generic yet practical three-round equivocal (string) commitment scheme under any OWF admitting Σ-protocols, which is depicted in Figure 1 (page 11).

The commitment stage of the protocol depicted in Figure 1 runs in 4 rounds, but it can be reduced into 3 rounds by merging Phase-2 into the second round of Phase-1. As we shall see, 3 rounds is the optimal round complexity for (black-box) equivocal commitments in the standard model.

Theorem 3.1 *Suppose f_V be* any *one-way function that admits Σ-protocols, the protocol depicted in Figure 1 is a three-round perfectly-hiding equivocal (string) commitment scheme in the standard model.*

Proof (of Theorem 3.1).

Perfect hiding.

The perfectly-hiding property of the protocol can be easily checked by noting that Phase-1 is independent of the message m to be committed and Phase-2 also perfectly hides m due to the *perfect* SHVZK property of Σ-protocols as discussed in Section 2.

Computational binding.

Note that the binding property of the protocol relies on the secrecy of the preimages of (y_V^0, y_V^1) and V does prove to P such knowledge in Phase-1. But the Σ_{OR}-protocol used in Phase-1 only guarantees WI property which is a much weaker security notion than zero-knowledge. And thus, one may argue that by interacting with V, an adversary P^* could potentially gain some knowledge about the preimages and the gained knowledge could help it violate the binding property. What save us here are the key pair trick (which is originally introduced by Naor and Yung in the PKE setting [55] and is also employed in the ZK setting [27]) and the (perfect) WI property of Σ_{OR}-protocols.

Specifically, suppose the protocol does not satisfy the binding property, then there exists a PPT adversary P^* such that with non-negligible probability $q(n)$ P^* can decommit the transcript of the commitment stage to two different values. Then we can construct a PPT algorithm E that breaks the one-wayness of f_V with non-negligible probability. On an input y in the range of f_V, E works as follows: It randomly selects an element x' of length n in the domain of f_V, computes $y' = f_V(x')$, randomly selects a bit b in $\{0, 1\}$, sets $y^b = y$ and $y^{1-b} = y'$. Finally, E sends (y^0, y^1) to P^*, and proves to P^* that it knows either the preimage of y^0 or the preimage of y^1 by executing the Σ_{OR}-protocol with x' as its witness. Now, suppose P^* can decommit the transcript of the commitment stage to two different values with non-negligible probability $q(n)$, then according to the special soundness of Σ-protocols it means that P^* can output a preimage of y^b or y^{1-b} with probability $q(n)$. But because the perfect WI property of Σ_{OR}-protocols, we know with probability $\frac{1}{2}q(n)$ E will get the preimage of $y^b = y$, which violates the one-wayness of f_V.

Equivocal commitments.

We directly deal with the more complicated three-round case when Phase-2 is combined into the second round of Phase-1. For any PPT adversary V^*, the PPT simulator S is depicted in Figure 2 (page 13).

Lemma 3.1 *S^c works in expected polynomial-time and the probability that S^c aborts with an error message is negligible.*

Proof (of Lemma 3.1). We charge each execution of Phase-1 of the protocol (depicted in Figure 1) as unit cost and our aim is to show that the expected charge accumulated in the **Repeat Loop** of Stage-2 is $poly(k)$.

Denote by p the probability that V^* successfully finishes Stage-1 in the simulation. As S^c goes into Stage-2 only if $p > 0$, below we distinguish two cases: $0 < p < 2^{-(k-1)}$ and $p \geq 2^{-(k-1)}$. When $0 < p < 2^{-(k-1)}$, the repeat loop is iterated at most $k \cdot 2^k < \frac{2k}{p}$ times. When $p \geq 2^{-(k-1)}$, each iteration of the repeat loop extracts a preimage of $(y^0_{V^*}, y^1_{V^*})$ with probability at least $p - 2^{-k}$. Thus, the expected number of iterations of the repeat loop in this case is less than $(p - 2^{-k})^{-1} \leq \frac{2}{p}$. Furthermore, as we shall see, with probability at least $1 - 2^{-k}$, the repeat loop is not repeated more than $\frac{2k}{p}$ times.

S^c may abort with an error messages only in two cases. The first case is when $p \geq 2^{-(k-1)}$, but in this case abort happens with probability at most $(1 - \frac{p}{2})^{k \cdot 2^k} < 2^{-k}$, since $k \cdot 2^k \geq \frac{2k}{p}$. The second case is when $0 < p < 2^{-(k-1)}$, but in this case S^c goes into Stage-2 only with probability $p < 2^{-(k-1)}$. We conclude that S^c aborts with an error message with probability at most $2^{-(k-1)}$. □

There are two differences between the simulated transcript outputted by S and the real interaction transcript between V^* and the honest P who is committing to a message s of length k. One is that in real interactions, P commits s and then reveals s accordingly, but in the simulation S always commits 0^k in the commitment stage and then reveals s in the decommitment stage; The second is that in the simulation S may abort with an error message. But, the first difference makes no distinction due to the perfectly-hiding property of the protocol (i.e., the perfect SHVZK property of Σ-protocols), and the second difference occurs only with negligible probability and thus also cannot make distinguishable distinction.

For the equivocability property, according to Definition 3.1, $p_1 = p$, and $0 \leq p_0 \leq p$ when $p < 2^{-(k-1)}$ or $(1 - 2^{-k}) \cdot p \leq p_0 \leq p$ when $p > 2^{-(k-1)}$. We conclude $|p_0 - p_1|$ is negligible. □

The equivocability simulator $S = (S^c, S^d)$

The simulation of S^c: S^c runs V^* as a subroutine and goes through the following (at most) two stages:

Stage-1. Denote by a, e, z the first, the second and the third message of the Σ_{OR}-protocol in Phase-1 of the commitment protocol (depicted in Figure 1) with respect to a pair $(y^0_{V^*}, y^1_{V^*})$. S^c runs V^* as a subroutine and in the second round (of the *combined* three-round protocol) it does the following: S^c runs $S_{OR}((y^0_{V^*}, y^1_{V^*}), 0^k)$ to get a simulated transcript, denoted $((\hat{a}_0, \hat{a}_1), 0^k, (\hat{e}_0, \hat{z}_0, \hat{e}_1, \hat{z}_1))$ (that is, S^c tries to commit 0^k), randomly selects a random string e in $\{0,1\}^k$ and sends $(e, (\hat{a}_0, \hat{a}_1))$ to V^*. Whenever V^* aborts with an incomplete execution of Phase-1 or (a, e, z) is not an accepting conversation with respect to $(y^0_{V^*}, y^1_{V^*})$, then S^c stops and outputs the transcript up to now. Otherwise, it goes to Stage-2.

Stage-2. Whenever S^c receives z from V^* in Stage-1 and (a, e, z) is an accepting conversation with respect to $(y^0_{V^*}, y^1_{V^*})$, S^c does the following:

Repeat Loop: Repeat up to $k \cdot 2^k$ times

- S^c rewinds V^* to the point that V^* just sent a, runs $S_{OR}((y^0_{V^*}, y^1_{V^*}), 0^k)$ (with fresh randomness) to get a simulated transcript, denoted $((\hat{a}'_0, \hat{a}'_1), 0^k, (\hat{e}'_0, \hat{z}'_0, \hat{e}'_1, \hat{z}'_1))$, randomly selects e' from $\{0,1\}^k \setminus \{e\}$ and returns back $(e', (\hat{a}'_0, \hat{a}'_1)))$ to V^*.
- S^c runs V^* further and if it receives back a z' such that (a, e', z') is an accepting conversation with respect to $(y^0_{V^*}, y^1_{V^*})$ (this means E gets the preimage of either $y^0_{V^*}$ or $y^1_{V^*}$) then S^c outputs $((y^0_{V^*}, y^1_{V^*}), a, e', z', (\hat{a}'_0, \hat{a}'_1))$ and stops. Otherwise, goto **Repeat Loop**.

Failure with Error Message. In case all the $k \cdot 2^k$ attempts have failed, S^c aborts with an error message.

The simulation of S^d: Denote by α the output of S^c. In case S^c did not stop in Stage-1 and did not abort with an error message in Stage-2, then for any string s of length k S^d does the following: it generates the decommitment information, denoted by $(\hat{e}''_0, \hat{z}''_0, \hat{e}''_1, \hat{z}''_1)$, by using the extracted preimage of either $y^0_{V^*}$ or $y^1_{V^*}$ as its trapdoor such that $((\hat{a}'_0, \hat{a}'_1), s, (\hat{e}''_0, \hat{z}''_0, \hat{e}''_1, \hat{z}''_1))$ constitute a successful conversation for showing the knowledge of the preimage of either $y^0_{V^*}$ or $y^1_{V^*}$.

Figure 2. The equivocability simulator

3.4 Optimal round-complexity

Finally, we show that the round-complexity (i.e., three rounds) of our generic yet practical equivocal commitment scheme is optimal. Specifically, we show that there is no 2-round (black-box) equivocal commitment scheme *of non-interactive decommitment stage* (that is the normal case of commitment schemes).

Theorem 3.2 *There is no 2-round (black-box) equivocal commitment scheme (of non-interactive decommitment stage). Note that this theorem holds* unconditionally.

Proof (of Theorem 3.2). We prove this theorem separately according to whether the prover or the verifier sends the first-round message. Specifically, we prove the following two propositions:

Proposition 3.1 *There is no 2-round (black-box) equivocal commitment scheme in which the verifier sends the first-round message.*

Proof (of Proposition 3.1). For any 2-round commitment scheme, denote by α (sent by the verifier) the first-round message and by β (sent by the prover) the second-round message. By the binding property, there is no PPT algorithm that *given* (a correctly generated) α can generate a valid β such that it can decommit (α, β) to two different values. This means that even if the commitment receiver always uses the same α in all its interactions (i.e., the commitment receiver correctly generates α and then fixes it once and for all) and even just publishes α as its public-key, still no PPT algorithm can violate the binding property with respect to the same α. Now, suppose the 2-round scheme is a equivocal commitment scheme, then according to the definition of equivocal commitments there is a PPT simulator that given the *fixed* α can generate a valid β such that it can decommit (α, β) to two different values. This violates the binding property of commitments. □

Proposition 3.2 *There is no 2-round (black-box) equivocal commitment scheme in which the prover sends the first-round message.*

Proof (of Proposition 3.2). Suppose there exists a 2-round black-box equivocal commitment scheme, then by the results of [48] that commitments imply OWF, we conclude that there exists a OWF. In other words, there exists an $\mathcal{NP}$-language L_{OWF} specified by the OWF such that $L_{OWF} \notin \mathcal{BPP}$. That is, $\mathcal{NP} \nsubseteq \mathcal{BPP}$.

Now, suppose there exists a 2-round black-box equivocal commitment scheme in which the prover sends the first-round message, we will construct a three-round black-box ZK argument for $\mathcal{NP}$. As only languages in $\mathcal{BPP}$ have three-round black-box ZK arguments [36], this implies that $\mathcal{NP} \subseteq \mathcal{BPP}$ (i.e., no OWF exists), which contradicts the above observation that commitments imply OWF.

Specifically, consider the Blum's protocol for the $\mathcal{NP}$-Complete language Directed Hamiltonian Cycle DHC (that is depicted in Appendix A). We replace the underlying perfectly-binding commitment scheme used in the first-round of Blum's protocol by the assumed 2-round prover-initiated black-box equivocal commitment scheme. The resulting protocol still runs in three rounds. The black-box ZK property of the resulting protocol is direct from the simulation indistinguishability and equivocability property of the assumed 2-round black-box equivocal commitment scheme. The computational soundness of the resulting protocol is from the computational binding property of the assumed 2-round equivocal commitment scheme. Specifically, the ability to answer two different challenges (sent in the second-round) with respect to the same equivocal commitments (executed in the first-round and a part of the second-round) implies breaking the computational binding property of the assumed 2-round equivocal commitment scheme. □ □

3.5 Implication and observation: Σ-protocols may witness non-$\mathcal{NP}$-Completeness

A list of long-standing and fundamental open problems, both in cryptography and complexity theory, are about the $\mathcal{NP}$-Completeness of many popular OWFs used in cryptography (e.g., Quadratic Residues, Factoring or RSA, etc) and some important problems in complexity theory (e.g., the Graph Isomorphism problem). Specifically, these problems are commonly believed to be not $\mathcal{NP}$-Complete, but developing formal proofs has been being the long-standing fundamental open problem in cryptography and complexity theory. In general, studying the (im)possibility of basing OWFs and/or public-key cryptography on $\mathcal{NP}$-Completeness is a puzzling question of fundamental nature [6, 32, 35, 5, 1].

In this work, we identify that the recent advances on statistical ZK and commitments may shed new light on these fundamental open problems. The reasoning is as follows:

Recall that, motivated to investigate the seemingly inherent (round) inefficiency of general complexity assumption based statistical commitments, it is very recently proved that non-constant round-complexity is intrinsic to *fully-black-box* constructions of statistical commitments from any one-way permutations (and even from trapdoor permutations) [44]. Intuitively, a fully-black-box reduction of a primitive P (e.g., statistical commitments in our case) to a primitive Q (e.g., OWF/OWP) is a construction of P out of Q that ignores the internal structure of the implementation of Q and just uses it as a "subroutine" (i.e., as a black-box). In addition, in the case of fully-black-box reductions, the proof of the security (showing that an adversary that breaks the implementation of P implies an adversary that breaks the implementation of Q), is also black-box (i.e., the internal structure of the adversary that breaks the implementation of P is ignored as well) [44]. The study of various black-box reductions between cryptographic primitives is originated from the seminal paper of Impagliazzo and Rudich [49], motivated to investigate the seemingly inherent limitation of basing some cryptographic primitives (e.g., key-agreement) on OWFs and to investigate the seemingly inherent inefficiency of OWF/OWP-based cryptographic primitives (with pseudorandom generator as a salient example). For precise definition of black-box reductions between cryptographic primitives, the reader is referred to [59].

Now, suppose there exists an $\mathcal{NP}$-Complete problem that admits Σ-protocols without intractability assumptions or based on general OWF/OWP assumptions. Then, by reducing any OWF/OWP to this $\mathcal{NP}$-Complete problem via $\mathcal{NP}$-reductions, and applying the construction of our statistical (equivocal) commitments from any OWF admitting Σ-protocols, we can get a three-round statistical (equivocal) commitments from any OWF/OWP. But, as the protocol construction and security analysis employ $\mathcal{NP}$-reductions, the assumed OWF-based statistical commitment protocol is not a black-box one. Nevertheless, in view of the seemingly inherent round inefficiency of general complexity assumption based statistical commitments [54, 45, 56, 46, 44], it naturally brings us to the conjecture:

Conjecture 3.1 *Any language admitting Σ-protocols (without intractability assumptions or based on OWF) is not $\mathcal{NP}$-Complete. In other words, a Σ-protocol is just a proof (witness) for showing that the language is not $\mathcal{NP}$-Complete.*

It would be significantly interesting to note that many popular OWFs (e.g., Quadratic Residues [42, 43]) and the Graph Isomorphism problem [33] indeed admit Σ-protocols *without intractability assumptions*, and the $\mathcal{NP}$-Completeness of them are long-standing open problems; On the other hand, it is indeed true that no known $\mathcal{NP}$-Complete language admits Σ-protocols (without intractability assumptions or based on OWF/OWP)[‡]. We hope that the above conjecture could be turned out to be true in the future, which would be a significant demonstration of the elegant interaction between cryptography and complexity theory.

4 Generic yet Practical Transformation from any Public-Coin HVZK Protocol to ZK Argument

The high-level overview of the transformation. Given a public-coin HVZK protocol $\langle P_L, V_L\rangle$ for a language L. Let $2t-1$ ($t \geq 2$) be the number of rounds of the protocol. Without loss of generality we assume all random challenges of the honest verifier V_L are of the same length. More precisely, the prover P_L is a PPT interactive machine that on input a string $x \in L$ of length n, an auxiliary input w, and a partial transcript T where T is either an empty string or a sequence of messages $(\alpha_1, c_1, \cdots, c_i)$, $1 \leq i \leq t-1$, outputs the next message α_{i+1}. The verifier algorithm V_L answers each prover message α_i with a challenge c_i of length k (that is polynomially related to n) taken uniformly at random from $\{0,1\}^k$, and at the end of interactions applies a verification procedure $V_L(x, \alpha_1, c_1, \cdots, \alpha_{t-1}, c_{t-1}, \alpha_t)$ to determine whether to accept or reject x. We also denote by S_L the HVZK simulator of $\langle P_L, V_L\rangle$.

To transform $\langle P_L, V_L\rangle$ into a normal ZK protocol $\langle P, V\rangle$ for the same language L, we design a coin-tossing mechanism to set the random challenges of $\langle P_L, V_L\rangle$ jointly by the prover and the (possibly malicious) verifier, by employing the generic yet practical equivocal commitment scheme developed in Section 3. The observation here is that all these $t-1$ equivocal commitments can share the same Phase-1 of the commitment stage (as described in Figure-1), and thus the incurred additional round-complexity could be minimal. The transformed protocol $\langle P, V\rangle$ is depicted in Figure 3 (page 16).

Comments on the incurred round-complexity: It's easy to check that, in comparison with the starting public-coin protocol $\langle P_L, V_L\rangle$, the number of the incurred additional rounds is that of Phase-1. Furthermore, the second-round and the third-round of Phase-1 can be merged into the first two rounds of Phase-2. Thus, the transformation only incurs additionally one round. (For public-coin HVZK protocols of even number of rounds, the transformation incurs additionally two rounds.)

[‡] It is interesting to note that Blums's protocol for DHC [4] is of *computational* ZK (based on general complexity assumption), but not statistical ZK as required by Σ-protocols; Note also that Blum's protocol for DHC can be converted into statistical HVZK argument, *but under specific complexity assumption* (by using collision-resistant hashing based non-interactive statistical commitments [22, 47] in the first-round).

The transformed ZK argument $\langle P, V\rangle$

Common input. An element $x \in L \cap \{0,1\}^n$.

P private input. An $\mathcal{NP}$-witness w for $x \in L$.

Phase-1. On a security parameter n, V selects a OWF f_V that admits Σ-protocols, randomly selects two elements in the domain of f_V, x_V^0 and x_V^1 of length n, and computes $y_V^0 = f_V(x_V^0)$ and $y_V^1 = f_V(x_V^1)$. Finally, V sends (y_V^0, y_V^1) to P, and proves to P that it knows the preimage of either y_V^0 or y_V^1 by executing the Σ_{OR}-protocol on (y_V^0, y_V^1) and playing the role of the knowledge prover. We denote by S_{OR} the perfect SHVZK simulator of the underlying Σ_{OR}-protocol. The witness used by V during the execution of the Σ_{OR}-protocol is x_V^b for a randomly chosen bit b in $\{0,1\}$. If V successfully finishes the Σ_{OR}-protocol, P moves into Phase-2; otherwise P aborts.

Phase-2. For $i := 1$ to t do: (Step-3 and Step-4 below can be merged with Step-1 and Step-2 of the following iteration.)

Step-1. If $i = t$ then P runs P_L to compute $\alpha_t = P_L(x, w, c_1, \cdots, c_{t-1})$, sends α_t to V, and the protocol $\langle P, V\rangle$ goes to "**Verifier's decision**". If $i \leq t-1$, the prover P randomly selects $c_P^{(i)}$ of length k, and runs the perfect SHVZK simulator $S_{OR}((y_V^0, y_V^1), c_P^{(i)})$ (as described in Section 2) to get a simulated transcript, denoted $((\hat{a}_0^{(i)}, \hat{a}_1^{(i)}), c_P^{(i)}, (\hat{e}_0^{(i)}, \hat{z}_0^{(i)}, \hat{e}_1^{(i)}, \hat{z}_1^{(i)}))$, pretending that it "knows" the preimage of either y_V^0 or y_V^1. Then, P runs P_L to compute $\alpha_i = P_L(x, w, c_1, \cdots, c_{i-1})$, where c_0 is set to be an empty string. Finally, P sends $((\hat{a}_0^{(i)}, \hat{a}_1^{(i)}), \alpha_i)$ to V.

Step-2. The verifier V sends back P a random challenge $c_V^{(i)}$ of length k.

Step-3. The prover P computes $c_i = c_P^{(i)} \oplus c_V^{(i)}$ and sends $(c_P^{(i)}, (\hat{e}_0^{(i)}, \hat{z}_0^{(i)}, \hat{e}_1^{(i)}, \hat{z}_1^{(i)}))$ to the verifier V.

Step-4. The verifier V checks that whether or not $((\hat{a}_0^{(i)}, \hat{a}_1^{(i)}), c_P^{(i)}, (\hat{e}_0^{(i)}, \hat{z}_0^{(i)}, \hat{e}_1^{(i)}, \hat{z}_1^{(i)}))$ is an accepting conversation for showing the knowledge of the preimage of either y_V^0 or y_V^1. If it is accepted then V computes $c_i = c_V^{(i)} \oplus c_P^{(i)}$, otherwise V aborts.

Verifier's decision The verifier accepts x if and only if $V_L(x, \alpha_1, c_1, \cdots, \alpha_{t-1}, c_{t-1}, \alpha_t)$ outputs 1.

Figure 3. The generic yet practical transformation from public-coin HVZK to (normal) ZK argument.

Theorem 4.1 *Assuming f_V be any OWF admitting Σ-protocols, the transformed protocol $\langle P, V\rangle$ (depicted in Figure 3) is a ZK argument for (the same) language L.*

Proof (of Theorem 4.1).

Zero-knowledge.

The strategy of the ZK simulator S is as follows: it first runs the HVZK simulator $S_L(x)$ (of the starting public-coin HVZK protocol $\langle P_L, V_L\rangle$) to get a simulated transcript $(\alpha_1, c_1, \cdots, \alpha_{t-1}, c_{t-1}, \alpha_t)$; Then, by running the malicious V^* as a subroutine, S tries to mimic the simulated transcript in the Phase-2 interactions (in case V^* successfully finishes Phase-1), by setting the outcome of the underlying coin-tossing to be c_i for each i, $1 \leq i \leq t-1$. Then, the ZK property can be easily derived from the simulation indistinguishability and equivocability of the underlying (generic yet practical) equivocal commitment scheme. The fact that our transformation preserves statistical/perfect ZK is from the observation that, according to the analyses of the simulation procedure of the equivocability simulator $S = (S^c, S^d)$ presented in the proof of Theorem 3.1, except for exponentially negligible probabilities the ZK simulator generates exactly the correct conversation.

Computational soundness.

Note that if the underlying equivocal commitment scheme (for coin-tossing) was perfectly-binding, the soundness of the transformed protocol $\langle P, V\rangle$ would trivially follows from the soundness of the starting public-coin HVZK protocol $\langle P_L, V_L\rangle$. This is the case in [52], where the nicely explored DDH-based

```
The algorithm $E(1^n, y)$

$x' \leftarrow \{0,1\}^n$; $y' \leftarrow f_V(x')$.
$b \leftarrow \{0,1\}$.
Set $y_V^b$ be $y'$, $x_V^b$ be $x'$ and $y_V^{1-b}$ be $y$.
$i \leftarrow \{1, 2, \cdots, t-1\}$.
$S$ runs $P^*$ as a subroutine and acts accordingly by playing the role of the honest verifier, until $E$ receives from $P^*$ the message $(c_{P^*}^{(i)}, (\hat{e}_0^{(i)}, \hat{z}_0^{(i)}, \hat{e}_1^{(i)}, \hat{z}_1^{(i)}))$. Note that in the interactions of Phase-1, $E$ uses $x' = x_V^b$ as its witness.
If $(c_{P^*}^{(i)}, (\hat{e}_0^{(i)}, \hat{z}_0^{(i)}, \hat{e}_1^{(i)}, \hat{z}_1^{(i)}))$ is valid (i.e., $P^*$ correctly decommits $(\hat{a}_0^{(i)}, \hat{a}_1^{(i)})$ with respect to the random challenge $c_V^{(i)}$ sent by $E$), then $E$ rewinds $P^*$ to the state that $P^*$ just sent $(\hat{a}_0^{(i)}, \hat{a}_1^{(i)})$.
$c_V^{(i)\prime} \leftarrow \{0,1\}^k \setminus \{c_V^{(i)}\}$.
Returns back $c_V^{(i)\prime}$ to $P^*$ and runs $P^*$ further, looking forward to receiving again a valid decommitment message of the same $(\hat{a}_0^{(i)}, \hat{a}_1^{(i)})$ but to a different value $c_{P^*}^{(i)\prime} \neq c_{P^*}^{(i)}$ from which $E$ can extract the preimage of either $y_V^0$ or $y_V^1$.
```

Figure 4. The algorithm $E(1^n, y)$

(simulatable) commitment scheme is perfectly-binding. But, for our case, the underlying equivocal commitment scheme is actually *computational binding* (actually, it seems that any equivocal commitment scheme intrinsically could not be perfectly-binding). Also, for any starting public-coin HVZK protocol, we cannot count on the special soundness as guaranteed by Σ-protocols. Thus the soundness proof here is (a bit) more complicated and subtle.

The key observation here is that, conditioned on a malicious polynomial-time (w.l.o.g., deterministic) P^* can successfully convince the honest verifier V of a false statement "$x \in L$" while $x \notin L$, then there must exist an i, $1 \leq i \leq t-1$, a positive polynomial $q(\cdot)$ and two disjoint sets, denoted $\mathcal{S}$ and $\mathcal{S}'$, where $\mathcal{S} \subseteq \{0,1\}^k$ and $\mathcal{S}' \subseteq \{0,1\}^k$ and $|\mathcal{S}| \geq \frac{1}{q(n)} \cdot 2^k$ and $|\mathcal{S}'| \geq \frac{1}{q(n)} \cdot 2^k$, such that P^* can decommit $(\hat{a}_0^{(i)}, \hat{a}_0^{(i)})$ to a value, denoted $c_{P^*}^{(i)} (\in \{0,1\}^k)$, for all random challenges $c_V^{(i)}$'s fallen into the set $\mathcal{S}$, and decommit $(\hat{a}_0^{(i)}, \hat{a}_0^{(i)})$ to another different value, denoted $c_{P^*}^{(i)\prime} (\in \{0,1\}^k \setminus \{c_{P^*}^{(i)}\})$, for all random challenges $c_V^{(i)}$'s fallen into the set $\mathcal{S}'$. The underlying reason is that, if the above does not hold then the computational soundness of the protocol $\langle P, V \rangle$ just follows from the soundness of the starting public-coin HVZK protocol $\langle P_L, V_L \rangle$.

Now, suppose the transformed protocol $\langle P, V \rangle$ does not satisfy computational soundness, then we will use the above observation to reach a contradiction to the one-wayness of f_V. Specifically, we will construct a PPT algorithm E that on common input y outputs a preimage of y under f_V with non-negligible probabilities in polynomial-time. The algorithm E is depicted in Figure 4 (page 17).

Suppose P^* can convince the honest verifier of a false statement "$x \in L$" while $x \notin L$ with non-negligible probability $\frac{1}{p(n)}$ for some positive polynomial $p(\cdot)$, then it is easy to check that E will output a preimage of either y_V^0 or y_V^1 with probability at least $\frac{1}{t \cdot q(n) \cdot p(n)}$ in polynomial-time. Due to the (perfect) WI property of the Σ_{OR}-protocol executed in Phase-1, we conclude with probability at least $\frac{1}{2 \cdot t \cdot q(n) \cdot p(n)}$ E will output the preimage of $y = y_V^{1-b}$ in polynomial-time, which violates the one-wayness of f_V. □

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A Blum's protocol for DHC [4]

Blum's protocol for the $\mathcal{NP}$-Complete language DHC (Directed Hamiltonian Cycle) is n-parallel repetitions of Blum's basic protocol for proving the knowledge of Hamiltonian cycle on a given directed graph G [4]. The following is the description of Blum's *basic* protocol for DHC:

Common input. A directed graph $G = (V, E)$ with $q = |V|$ nodes.

Prover's private input. A directed Hamiltonian cycle C_G in G.

Round-1. The prover selects a random permutation, π, of the vertices V, and commits (using a perfectly-binding commitment scheme) the entries of the adjacency matrix of the resulting permutated graph. That is, it sends a q-by-q matrix of commitments so that the $(\pi(i), \pi(j))^{th}$ entry is a commitment to 1 if $(i, j) \in E$, and is a commitment to 0 otherwise.

Round-2. The verifier uniformly selects a bit $b \in \{0, 1\}$ and sends it to the prover.

Round-3. If $b = 0$ then the prover sends π to the verifier along with the revealing of all commitments (and the verifier checks that the revealed graph is indeed isomorphic to G via π); If $b = 1$, the prover reveals to the verifier only the commitments to entries $(\pi(i), \pi(j))$ with $(i, j) \in C_G$ (and the verifier checks that all revealed values are 1 and the corresponding entries form a simple q-cycle).

B Result Strengthening and Extension in the Current Version

In comparison with the original version of the ECCC report, the results presented in the current version are strengthened and extended in the following aspects:

1. The proof of Theorem 3.2 (for showing the round-complexity lower-bound of equivocal commitments) is made to be *unconditional*, by noting the result of [48] that commitments imply OWF. The proof in the original version is conditioned on $\mathcal{NP} \not\subseteq \mathcal{BPP}$.

2. We clarify the necessity of preimage-verifiable OWF in Damgård's paradigm of constructing perfectly-hiding trapdoor commitments with Σ-protocols. This implies, to the best of our knowledge, that our round-optimal equivocal commitment scheme is the *first* constant-round statistically-hiding (not necessarily to be trapdoor or equivocal) commitment scheme based on *any* OWF admitting Σ-protocols.

3. The implication that Σ-protocols may witness $\mathcal{NP}$-Completeness is clarified and observed in Section 3.5.

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