Regarding different approaches to the limits of OBDD proof systems

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Recently two papers have been announced that establish proof size lower bounds for propositional proof systems based upon ordered binary decision diagrams [1, 2]. The two papers were developed independently, announced on the proof complexity mailing list within hours of one another in January 2007, and appeared as ECCC tech reports shortly thereafter. Here, in a nutshell, are some relevant points of comparison:

1. The distinction between "exponential" and "nearly-exponential" in the titles of the two papers exists only in the choice of wording - both bounds are of the form $2^{N^{\Omega(1)}}$ (where N is the size of the input CNF) and neither bound is of the form $2^{\Omega(N)}$.
2. Krajíček's bound holds for the DAG-like OBDD system whereas my bound holds for only the tree-like OBDD system. I know of no OBDD-based satisfiability algorithms that construct DAG-like proofs, and finding a good heuristic for doing this could be very interesting.
3. For what it is worth, the analysis in [1] does not apply in any clear way to the CNF $IndMatch_m$ that is proved difficult for tree-like OBDD refutations in [2]. The analysis in [1] depends heavily on every variable being subject to the permutation (so that one can recover an OBDD refutation of the original formula with any desired variable ordering from an arbitrary OBDD refutation of modified formula), whereas in the formula $IndMatch_m$, not every variable is subject to the permutation and it does not seem likely that one can recover an OBDD refutation of $Match_m$ with any desired variable ordering from an arbitrary OBDD refutation of $IndMatch_m$.

References

[1] Jan Krajíček. An exponential lower bound for a constraint propagation proof system based on ordered binary decision diagrams. Technical Report TR07-007, Electronic Colloquium on Computational Complexity, 2007.

[2] N. Segerlind. Nearly exponential size lower bounds for symbolic quantifier elimination algorithms and OBDD-based proofs of unsatisfiability. Technical Report TR07-009, Electronic Colloquium on Computational Complexity, 2007.

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