Electronic Colloquium on Computational Complexity, Comment 1 on Report No. 72 (2009)

A note on the Raz-McKenzie method and the pattern matrix method

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Abstract

This short note relates and contrasts two methods in communication complexity, a method due to Raz and McKenzie [RM] and the pattern matrix method [S1, S2].

A method due to Raz and McKenzie [RM] and the pattern matrix method [S1, S2] are two techniques for proving communication lower bounds. What relates them is the kind of communication problem they apply to:

- In [RM], one fixes a DNF formula Φ that is identically true; Alice receives an n-element subset $S \subset \{1, 2, \ldots, N\}$; Bob receives a string $x \in \{0,1\}^N$; and the goal of the communication problem is to output a term of Φ satisfied by $x|_S$.
- In [S1, S2], one fixes a Boolean function $f\colon \{0,1\}^n \to \{0,1\}$; Alice receives an n-element subset $S \subset \{1, 2, \ldots, N\}$; Bob receives a string $x \in \{0,1\}^N$; and the goal of the communication problem is to compute $f(x|_S)$.

(The two definitions above leave out inessential detail.) The two works differ fundamentally as to the techniques used and results achieved. In particular:

- The Raz-McKenzie method is not known to generalize beyond the two-party deterministic model, whereas the pattern matrix method applies to randomized [S2], quantum [S2], weakly unbounded [S1, S2], and multiparty [C, LS, CA, DP, DPV, BH] communication complexity. On the other hand, neither method implies the other because the communication games are different; in particular, the Raz-McKenzie method optimally tackles problems in two-party deterministic complexity to which the pattern matrix method does not even apply.
- The techniques of the two works are unrelated: the method of [RM] is combinatorial, whereas the pattern matrix method [S1, S2] is analytic (based on linear programming duality).
- Accordingly, the communication lower bounds in [RM] are in terms of a *combinatorial* complexity measure (deterministic query complexity of Φ as a search problem), and those in [S1, S2] are in terms of *analytic* complexity measures (uniform approximation and sign-representation of f as a real function by polynomials).

The communication problems in [RM] and [S1, S2]—both based on the idea of creating a hard problem by applying the same function f to various subsets of the variables—have well-known earlier analogues in other computational models, including the Nisan-Wigderson generator [NW] and circuit lower bounds due to Krause and Pudlák [KP].

Acknowledgments

The author is thankful to Ran Raz for his time and feedback in the preparation of this note, as well as to the authors of [BHP] for a fruitful discussion shortly after the publication of [BHP] on ECCC and for pointing out [RM] in the first place.

ISSN 1433-8092

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ECCC ISSN 1433-8092

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