The assumption of Theorem 2 that 1-branching programs of width 3 need to be weakly oblivious can indeed be removed (cf. Footnotes 1 and 2 on p. 3 and p. 15, respectively). It follows from equations (64) and (65) that Theorem 3 holds for a stronger richness condition (cf. the original definition on p. 5): $A \subseteq \{0,1\}^*$ is strongly ε-rich if for sufficiently large n, for any index set $I \subseteq \{1,\ldots,n\}$, and for any partition $\{R_1,\ldots,R_r\}$ of I (where $r \geq 0$) the following implication holds: If $\prod_{j=1}^{r}(1-1/2^{|R_j|}) \geq \varepsilon$, then for any $c \in \{0,1\}^n$ and for any $Q \subseteq \{1,\ldots,n\}\setminus I$ satisfying $|Q| \leq \log n$ there exists $a \in A \cap \{0,1\}^n$ such that $a_i = c_i$ for every $i \in Q$, and for every $j \in \{1,\ldots,r\}$ there exists $i \in R_j$ such that $a_i \neq c_i$.

This stronger richness condition can then be employed in the proof of Theorem 2 for non-oblivious width-3 1-branching programs as follows. Since (non-oblivious) P is read-once, the classes R_b (see the definition in Paragraph 5.1) are pairwise disjoint for different blocks b except for the special case of $m_{b-1} = \nu_b$ for non-empty block $b > 1$. In this special case, we know $q_b = 0$ (i.e. no Q_{bj} is defined for block b) and either $t_{12}^{(m_{b-1})} = t_{32}^{(m_{b-1})} = \frac{1}{2}$ and $t_{33}^{(m_{b-1})} = 1$ if $\gamma_b = \nu_b$ or $t_{13}^{(m_{b-1})} = t_{33}^{(m_{b-1})} = \frac{1}{2}$ and $t_{32}^{(m_{b-1})} = 1$ if $\gamma_b < \nu_b$ (cf. the sentence following equation (31) on p. 19). Thus, index $i \in R_b$ of the variable that is tested either at node $v_2^{(m_{b-1}-1)}$ if $\gamma_b = \nu_b$ or at node $v_3^{(m_{b-1}-1)}$ if $\gamma_b < \nu_b$ may possibly be included also in R_{b-1}. In order to secure that R_b and R_{b-1} are disjoint we redefine $R'_b = R_b \setminus \{i\}$ in this special case, which replaces $|R|$ with $|R|+1$ in equation (32) while inequality (37) remains still valid. This ensures that the classes R_b are pairwise disjoint also for non-oblivious P.

For the recursive step in Paragraph 7.2, the stronger richness condition for $Q = \emptyset$ coincide with the original one. In the end of recursion (Section 8), on the other hand, inequality (61) ensures there is $Q = Q_{b^*j^*}$ for some $b^* \in \{1,\ldots,r+1\}$ and $j^* \in \{1,\ldots,q_{b^*}\}$ such that $|Q| \leq \log n$ according to (64), and the stronger richness condition can be employed for Q and $R_1,\ldots,R_{b^*-1}$ according to (40), provided that $R_b \cap Q = \emptyset$ for every $b = 1,\ldots,b^*-1$. This disjointness follows from the fact that P is read-once except for the special case of $R_{b^*-1} \cap Q = \emptyset$ for $j^* = 1$, $\kappa_{b^*1} = \sigma_{b^*1} = m_{b^*-1}$, and $t_{23}^{(m_{b^*-1})} = 0$ (see the definition of Q in Paragraph 5.2). In this particular case, however, it clearly suffices to use the stronger richness condition for $R_1,\ldots,R_{b^*-2}$ and Q.

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