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Rate One-Third Non-malleable Codes

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Abstract

At ITCS 2010, Dziembowski, Pietrzak and Wichs introduced Non-malleable Codes (NMCs) which protect against tampering of a codeword of a given message into the codeword of a related message. A well-studied model of tampering is the 2-split-state model where the codeword consists of two independently tamperable states. As with standard error-correcting codes, it is of great importance to build codes with high rates.

Following a long line of work, Aggarwal and Obremski (FOCS 2020) showed the first constant rate non-malleable code in the $2-$split state model; however this constant was a minuscule 10^{-6}! In this work, we build a Non-malleable Code with rate $1/3$. This nearly matches the rate $1/2$ lower bound for this model due to Cheraghchi and Guruswami (ITCS 2014). Our construction is simple, requiring just an inner-product extractor, a seeded extractor, and an affine-evasive function.

1 Introduction

The security of cryptographic primitives, often times, hold only under idealized assumptions. If we take the case of encryption or digital signatures (or most other primitives), the definition of security follows a standard template: the adversary gets to observe input-output behaviour and then needs to attack the primitive. In most cases, the implicit assumption is that the adversary gets to learn nothing other than this legitimate input-output behaviour. There are many instances where such an idealized setting is far from reality. For instance, there is a powerful class of attacks where the adversary could tamper with the secret keys of the system and observe input-output behaviour on these tampered keys. Such attacks can completely break the security of the system [BS97, AARR02, LL10].

Motivated by the need to protect data against such tampering attacks, Dziembowski, Peitrzak and Wichs, in their pioneering work, introduced "Non-malleable Codes" (NMCs). Non-malleability is a widely studied notion in cryptography and strives to defend against *related* tampering. Non-malleable codes, in specific, are coding schemes where the adversary cannot tamper the codeword into the codeword of a related message. Informally, NMCs provide the following guarantee: given

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a codeword C of a message m and a function f chosen from a tampering family $\mathcal{F}$, decoding $f(C)$ gives something that is either equal to m or completely independent of m. This becomes extremely relevant when NMCs are used to store the secret key. NMCs assure us that the encoding of the secret key may be tampered with, but the tampered codeword decodes to a message that is independent of the original secret key. A consequence of this for the above-mentioned scenario is that observing input-output behaviour on this tampered key is now rendered useless and the cryptosystem continues to remain secure.

As observed in [DPW10], it is impossible to build NMCs secure against unrestricted tampering i.e., when $\mathcal{F}$ corresponds to the family of all functions. (To see this, simply consider the function $f(c) = \mathsf{Enc}(\mathsf{Dec}(c) + 1)$.) NMCs are therefore built with respect to specific classes of functions. A well-studied class of tampering functions is the 2-split-state model where the codeword consists of two states L and R, and the adversary tampers with each of these states independently. This is a very natural model and indeed such NMCs have found many applications in securing against physical (leakage and tampering) attacks [DPW10, LL12], domain extension of encryption schemes [CMTV14, CDTV16], non-malleable commitments [GPR16], non-malleable secret sharing [GK18, ADN$^+$19, BS19, SV19] and privacy amplification [CKOS19].

As with standard error correcting codes, the most important parameter for a non-malleable code is its rate $= \frac{\kappa}{n}$ where κ denotes the message length and n the codeword length. Cheraghchi and Guruswami [CG14a] showed that the optimal achievable rate for the 2-split-state family is $\frac{1}{2}$. The quest for this holy grail has inspired a long line of fascinating research, introduced new pseudorandom objects and intricate proof techniques. Most recently, Aggarwal and Obremski [AO20] constructed a constant rate 2-split-state NMC (with negligible error). While asymptotically significant, they achieve a constant rate of only around $1/1,500,000$. Thus, there is an embarrassingly large gap between the best achievable rate $(1/2)$ and what we currently know $(1/1,500,000)$! *In this work, we build 2-split state non-malleable codes with a near-optimal rate* $\frac{1}{3}$.

1.1 Our Results

We obtain our construction of a rate-$\frac{1}{3}$ NMC via rate boosters which compile low rate NMCs into high rate ones. Using similar, but simpler, techniques we also obtain rate boosters for related primitives called "non-malleable 2-source extractors" ($\mathsf{nm2Ext}$). We start by explaining our results on $\mathsf{nm2Ext}$ before proceeding to describe our results on NMCs.

Towards the goal of constructing non-malleable codes, Cheraghchi and Guruswami [CG14b] introduced Non-malleable Extractors as a stronger primitive that immediately yields efficient non-malleable codes as long as the preimage of the extractor is efficiently samplable. Informally, a non-malleable two source extractor $\mathsf{nm2Ext}$ guarantees that for any independent random sources X, Y, and any functions f, g with at least one of them having no fixed points, $\mathsf{nm2Ext}(X, Y)$ is indistinguishable from uniform even given $\mathsf{nm2Ext}(f(X), g(Y))$.[1] It is easy to see that a non-malleable two-source extractor gives non-malleability for a uniformly random message (average-case security) while a non-malleable code achieves non-malleability for every message (worst-case security). A non-malleable two-source extractor can be transformed into a non-malleable code $(\mathsf{Enc}, \mathsf{Dec})$ by setting $\mathsf{Enc}(m) := \mathsf{nm2Ext}^{-1}(m)$, and $\mathsf{Dec}(x, y) := \mathsf{nm2Ext}(x, y)$. Non-malleable $2-$source extractors have been the focus of intense study, with several recent results focusing on improving their rate.

Rate Boosters for Non-malleable Extractors. In this work, we give a rate booster for unbal-

[1] We say that the extractor is a strong non-malleable two-source extractor if for any independent random sources X, Y, and any functions f, g with at least one of them having no fixed points, $\mathsf{nm2Ext}(X, Y)$ is indistinguishable from uniform even given $\mathsf{nm2Ext}(f(X), g(Y))$ and Y.

anced non-malleable two-source extractors, with one source being much smaller than the other:

Informal Theorem 1. *If* $\mathsf{nm2Ext} : \{0,1\}^{n_1} \times \{0,1\}^{n_2} \to \{0,1\}^d$ *is a strong non-malleable two-source extractor, with* $n_2 = o(n_1)$*, and* $\mathsf{Ext} : \{0,1\}^{n_1} \times \{0,1\}^d \to \{0,1\}^{\frac{n_1}{2}}$ *is a strong seeded extractor, then* $\mathsf{nm2Ext}'(x,y) := \mathsf{Ext}(x; \mathsf{nm2Ext}(x,y))$ *is a non-malleable extractor with rate* $1/2$*.*

NME to NMC: Limitations. We note that the transformation from nm2Ext to NMCs requires arguing worst-case security from average-case security, which incurs a factor $2^{|\mathsf{message\ size}|}$ penalty in the security parameter. Most results on building 2-split-state NMCs have focused on improving the rate of non-malleable two-source extractors and relied on this *lossy* transformation to build NMCs. We argue that this may not help.

Note that the output of a seeded extractor, with seed length d, is at best $2^{-d/2}$-close to uniform. So we cannot use the transformation from non-malleable two-source extractors to non-malleable codes mentioned above, since our construction is unable to handle the penalty $2^{|\mathsf{message\ size}|} = 2^{n_1/2}$ in the error. (For the error term to even be just less than 1, $d > n_1/2$ i.e., a setting of parameters where the transformation above is useless.) We observe that this is not just a shortcoming of our construction – any construction of a non-malleable two-source extractor with rate greater than $1/5$ can likely not be transformed to a non-malleable code via the above average to worst-case transformation. To see this, we note that the existential construction via the probabilistic method (which is believed to have optimal parameters) of an ε-non-malleable two-source extractor given in [CG14b] (Theorem 5.10) requires $\varepsilon^3 \cdot 2^{2n} > 2^{2\kappa}$, where κ is the message length. This implies that $\varepsilon > 2^{(2\kappa-2n)/3}$. To obtain a non-trivial construction of a non-malleable code, we require $\varepsilon < 2^{-\kappa}$ which gives $2n > 5\kappa$, i.e., the rate of the non-malleable two-source extractor is less than $1/5$.

Our Non-malleable Code. With this intuition, we deviate entirely from the approach of building NMCs via Non-malleable 2-source Extractors. We significantly modify the transformation (in Informal Theorem 1) and introduce techniques that allow us to circumvent the average to worst case penalty to obtain a rate booster directly for non-malleable codes.

Informal Theorem 2 (Main Result). *There exists an efficient, information-theoretically secure* ε*-right-augmented*[2] *non-malleable code in the* 2*-split-state model with rate* $1/3$*. We show two instantiations of the scheme: the first gives us a strikingly simple construction (as we describe in Section 1.2) and achieves an error of* $2^{-\Omega(\kappa^{1/5}/\operatorname{polylog}(\kappa))}$*; the second instatiation loses out on the simplicity but achieves an error of* $\varepsilon = 2^{-\Omega(\frac{\kappa}{\log^3 \kappa})}$*, where* κ *is the size of the message.*

1.2 Technical Overview

In order to highlight the challenges of building high-rate, 2-split-state NMCs, we start by recalling a related primitive called Non-malleable Randomness Encoders introduced by Kanukurthi, Obbattu and Sekar [KOS18]. Similar to a 2-split-state NMC, a 2-split-state NMRE consists of two independently tamperable states L and R. Contrary to an NMC, where the encoder encodes arbitrary messages, an NMRE's encoder outputs L and R such that they decode to a random string, and

[2]Right-augmented property guarantees that the right state of the NMC is simulatable independent of the message, along with the tampered message.

herein lies all the difference: while the problem of building high-rate NMCs has eluded researchers for over a decade, we know how to build NMREs with rate $\frac{1}{2}$. At the same time, we emphasize that obtaining a high-rate NMC (instead of an NMRE) is critical for many applications (such as non-malleable commitments.)

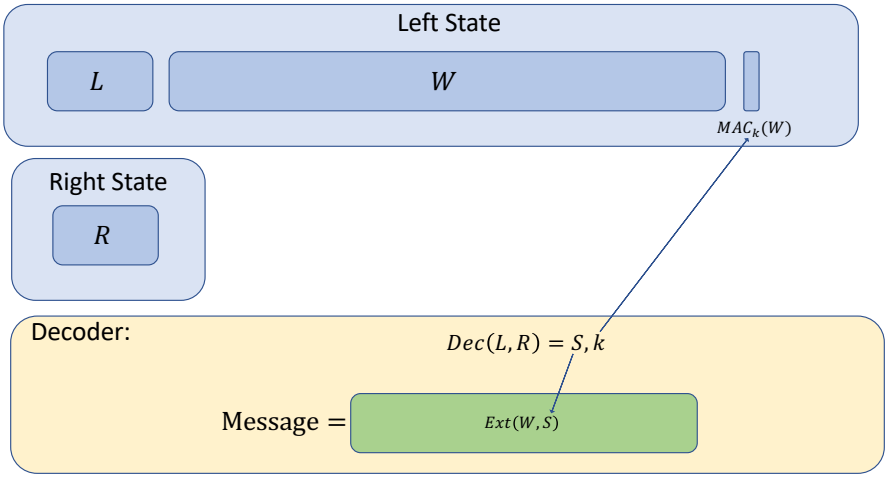

Figure 1: Construction of the non-malleable randomness encoder by [KOS18].

Informally, the NMRE of [KOS18] (see Figure 1.) picks the source w and seed s to a strong seeded extractor (Ext) as well as a key k_w to a message authentication code (MAC). The code consists of two states, left: $\ell \| w \| t$, and right: r, where ℓ, r are a low-rate non-malleable encoding of s, k_w and σ_w is a tag evaluated on w with k_w as the key. The codeword, if valid, decodes to $\mathsf{Ext}(w; s)$. The high level idea behind security is that if the codeword was tampered and leads to a different $\underline{s}$, then it is independent of S and extractor security applies. This suffices to argue independence of the tampered message $\underline{m}$ from m. (In the case where $\underline{s}, \underline{k_w} = s, k_w$, MAC security comes into play.)

In order to extend this construction to encode an arbitrary message m, one option would be to reverse sample w and s such that $\mathsf{Ext}(w; s) = m$. Unfortunately, this won't work because, on the one hand, we require the seed s to be short (as it is encoded using a poor-rate NMC) and, on the other hand, given a source w, there will be at most $2^{|s|}$ possible messages that could have been encoded. In other words, to obtain any meaningful security, s needs to be as long as the message. However, if s is long, the above approach will not yield rate gains. These orthogonal constraints are the main stumbling block which we overcome in this work. To do so, we need to somehow use $\mathsf{Ext}(w; s)$ to "mask" the message m. Clearly, since $\mathsf{Ext}(w; s)$ needs to be stored in a state different from the one storing w, we will consider a target code which has the form $(\ell \| w \| \sigma_w), (r \| c)$, where c would depend on the message as well as the extractor output $\mathsf{Ext}(w; s)$ i.e., w, s, c "shares" m. While there are many candidates for c, the solution to our puzzle lies in figuring out a c with which we can prove non-malleability. Here's the tricky part: when the adversary learns the tampered message, she learns information that depends on $\mathsf{Ext}(w; s)$. Furthermore, her tampering of r, which influences the tampered seed $\underline{s}$ and therefore $\mathsf{Ext}(\underline{w}; \underline{s})$, also depends on $\mathsf{Ext}(w; s)$. In a nutshell, regardless of what c is, it is clear that leveraging the security of the strong randomness extractor to argue non-malleability is not straight-forward.

Our first idea to overcome this challenge is to allow c to be such that $\mathsf{2Ext}(\mathsf{Ext}(w; s), c) = m$ where $\mathsf{2Ext}$ is a $2-$source extractor. As it turns out, this construction is secure for the following reason: even if the tampered output leaks some information about $\mathsf{Ext}(w; s)$, the two source ex-

tractor[3] will ensure that the "masking" remains secure. This leads to an NMC with rate $\frac{1}{9}$ in the split-state model.

The main idea behind our final construction is to tackle the information leakage on $\mathsf{Ext}(w;s)$ head-on. In particular, though the tampered message may leak information about $\mathsf{Ext}(w;s)$, we can ensure that this information is useless as far as breaking non-malleability is concerned. In fact, we do this while further simplifying our construction: We set the second half of the right state to be $(c = \mathsf{Ext}(w;s) \oplus m) \| \sigma_c$ where σ_c is a MAC tag of c evaluated on key k_c. In retrospect, our encoding scheme is nearly identical to that due to Kanukurthi, Obbattu and Sekar [KOS17]. While [KOS17] gave a four state construction, we merge states to obtain a two state construction.
We now offer a more detailed overview of the construction as well as the proof.

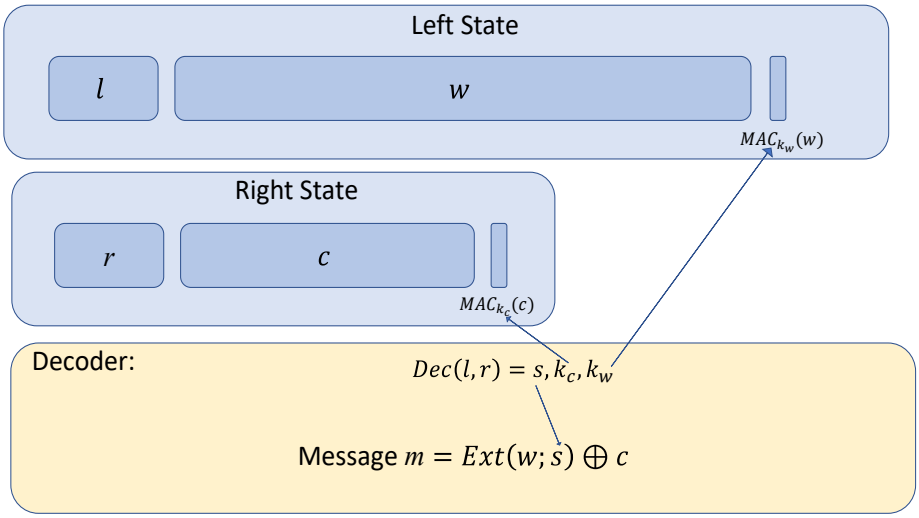

Figure 2: Overview of the construction. Blocks ℓ, r come from augmented non-malleable code. The encoder proceeds in steps: first, we randomly sample s, k_w, k_c, w (all independently of the message we are encoding), then we encode s, k_w, k_c using NMC into ℓ, r. We then set $c = \mathsf{Ext}(w;s) \oplus m$, and evaluate σ_c as a MAC tag of c on key k_c, and σ_w as MAC tag of w on key k_w.

Our construction uses the following building blocks: a message authentication code, a strong seeded extractor, and a low-rate non-malleable code which we shall use to encode the keys of the message authentication code and the seed for the seeded extractor. The overview of the construction can be found in the Figure 2. We will use the notation from the above-mentioned figure. Also, for a variable X, $\underline{X}$ will denote its tampering. We proceed with a slightly simplified sketch of the proof.

Proof Overview In order to prove non-malleability we need to demonstrate the existence of a simulator whose outputs is indistinguishable from the output of the tampering experiment. The simulator doesn't use the message; however, it outputs a special symbol to indicate that the tampered message is unchanged. The simulator's output is run through a special wrapper function (typically called "Copy" function) that, in this case, outputs the original message.

Our proof proceeds by partitioning our codeword space. Our simulator will pick a codeword at random, checks which partition it lies in to determine its output. We describe the partitions below:

[3] A two source extractor takes two entropic sources and outputs perfect randomness.

- $\mathcal{P}_1 = \{(\ell, w, \sigma_w, r, c, \sigma_c) : (w, \sigma_w, c, \sigma_c) = (\underline{w}, \underline{\sigma_w}, \underline{c}, \underline{\sigma_c}) \wedge \mathsf{NMDec}(\ell, r) = \mathsf{NMDec}(\underline{\ell}, \underline{r})\}$. This partition captures the scenario when, even after tampering, the inner codeword (ℓ, r) decodes to the same message, and W, σ_w, C, σ_c remain unchanged. In this case, the final codeword must decode to the same message.

- $\mathcal{P}_2 = \{(\ell, w, \sigma_w, r, c, \sigma_c) : (w, \sigma_w, c, \sigma_c) \neq (\underline{w}, \underline{\sigma_w}, \underline{c}, \underline{\sigma_c}) \wedge \mathsf{NMDec}(\ell, r) = \mathsf{NMDec}(\underline{\ell}, \underline{r})\}$. $\mathcal{P}_2$ captures the scenario when, the decoding of the inner code remains unchanged after tampering, while one of the pairs (W, σ_w) or (C, σ_c) are changed. We will show that if this event occurs then, using the security of MACs, the tampering is detected with overwhelming probability.

- $\mathcal{P}_3 = \{(\ell, w, \sigma_w, r, c, \sigma_c) : \mathsf{NMDec}(\ell, r) \neq \mathsf{NMDec}(\underline{\ell}, \underline{r})\}$. $\mathcal{P}_3$ captures the scenario that the inner code is non-trivially tampered and does not decode to $\mathsf{Dec}(L, R)$. In this case, we will show that the tampered codeword is independent of the original message m.

The simulator generates the codeword $((L, W, \sigma_w), (R, \widetilde{C}, \widetilde{\sigma}_c))$ of a random message. If this simulated codeword is in $\mathcal{P}_1$, it outputs same^*. Recall that the wrapper function will then output the original message. If the simulated codeword is in $\mathcal{P}_2$, the simulator outputs $\perp$, else the simulator outputs $\mathsf{Dec}\Big((L, W, \sigma_w), (R, \widetilde{C}, \widetilde{\sigma}_c)\Big)$. (Note that our code is right-augmented i.e., it satisfies a stronger notion of security where the right state of the codeword can be revealed without breaking non-malleability.)

To prove non-malleability, we need to show that this behaviour of the simulator is indistinguishable from that of the tampering experiment. To do this, we first show that the probability of a codeword being in any given partition is independent of the message[4]. Next, we show that the output of the tampering experiment is, in each case, indistinguishable from the simulator's output.

For the case where the codeword is in partition $\mathcal{P}_1$, it is clear that the simulator output is identical to that of the tampering experiment. We therefore focus on the other two cases.

1.2.1 Case 1: Codeword is in $\mathcal{P}_2$ i.e., $\mathsf{NMDec}(\underline{L}, \underline{R}) = \mathsf{NMDec}(L, R)$.

Intuitively, we would like to argue that the tag keys K_w, K_c will remain securely hidden from the adversary, and if he decides to tamper with W or C he will not be able to fake tags σ_w, σ_c. Thus either the whole codeword remains untampered (in which case, we are in $\mathcal{P}_1$) or the new codeword will not be valid.

The standard approach would be to argue that if $\Pr(\mathsf{NMDec}(\underline{L}, \underline{R}) = \mathsf{NMDec}(L, R))$ is not too small then

$$\Pr(\text{tampered codeword is valid } \wedge (\underline{W}, \underline{C}) \neq (W, C) \mid \mathsf{Dec}(\underline{L}, \underline{R}) = \mathsf{Dec}(L, R))$$

is negligible. However we have to be delicate here. For example if adversary wants to tamper with W he has access to L and knows that $\mathsf{NMDec}(\underline{L}, \underline{R}) = \mathsf{NMDec}(L, R)$. This reveals some information about R and thus adversary potentially might get hold of some partial information about the encoded data (and K_w, K_c in particular). This is why it is actually easier to directly argue that

$$\Pr(\text{tampered codeword is valid } \wedge (\underline{W}, \underline{C}) \neq (W, C) \wedge \mathsf{Dec}(\underline{L}, \underline{R}) = \mathsf{Dec}(L, R)) \tag{1}$$

is negligible. Notice that the codeword will not be valid in only one of three cases: if $\mathsf{NMDec}(\underline{L}, \underline{R}) = \perp$ or if one of the MACs on W or C does not verify correctly. Since $\mathsf{NMDec}(\underline{L}, \underline{R}) = \mathsf{NMDec}(L, R)$

[4]This proof relies on the secret sharing property of the non-malleable code as well as the security of the strong randomness extractor.

we know that the only options left are the failures to verify MACs. Moreover we know that $(\underline{K_w}, \underline{K_c}) = (K_w, K_c)$, thus Inequality 1 can be rewritten:

$$\Pr(\mathsf{Vrfy}_{K_w}(W, \sigma_w) = \mathsf{Vrfy}_{K_c}(C, \sigma_c) = 1 \wedge (\underline{W}, \underline{C}) \neq (W, C) \wedge \mathsf{NMDec}(\underline{L}, \underline{R}) = \mathsf{NMDec}(L, R)) \quad (2)$$

is negligible. Now we can upper-bound the term in the Inequality 2 by the following

$$\Pr(\mathsf{Vrfy}_{K_w}(W, \sigma_w) = \mathsf{Vrfy}_{K_c}(C, \sigma_c) = 1 \wedge (\underline{W}, \underline{C}) \neq (W, C)).$$

Which by the union bound can be upper-bounded with

$$\Pr(\mathsf{Vrfy}_{K_w}(W, \sigma_w) = 1 \wedge \underline{W} \neq W) + \Pr(\mathsf{Vrfy}_{K_c}(C, \sigma_c) = 1 \wedge \underline{C} \neq C).$$

Finally we can argue that each of elements of the sum is negligible. Notice that when tampering with W adversary has access to L but that can not reveal any information about K_w since every non-malleable code is a secret sharing scheme. The rest follows from the security of MACs.

1.2.2 Case 2: Codeword is in $\mathcal{P}_3$ i.e., $\mathsf{NMDec}(\underline{L}, \underline{R}) \neq \mathsf{NMDec}(L, R)$.

In this case, we will follow the adventures of the seed S; the MACs and keys do not play any role here. In fact, for the purposes of this proof sketch we will ignore the MAC keys and tags. We will also assume that this case (i.e., codeword $\in \mathcal{P}_3$) occurs with substantial probability (else we don't have to worry about it). In such a case, we will argue that the final message is independent of the original message.

To provide a proof sketch for this case, we consider the following modified construction: Let $(\ell, r) \leftarrow \mathsf{NMEnc}(s)$, where $(\mathsf{NMEnc}, \mathsf{NMDec})$ is the low-rate non-malleable code. Pick a random W, the source for a strong extractor. Then the left state of the final non-malleable code is $\ell \| w$. The right state is $r \| (c = \mathsf{Ext}(w; s))$. In other words, we've simply removed the MAC keys and tags. While this is obviously not a secure non-malleable code, we use this construction for the sake of ease of exposition.

By the properties of the augmented NMC[5] we know that there is a simulator NMSim that outputs $S^{\mathsf{sim}}, L^{\mathsf{sim}}$. Given that we are in the case where $\mathsf{NMDec}(\underline{L}, \underline{R}) \neq \mathsf{NMDec}(L, R)$, S^{sim} is independent of S. As discussed earlier, this knowledge alone doesn't suffice for us. The tampering of R (which influences $\underline{S}$) depends on C which in turn uses the extractor output. In order to output the tampered message, we need to be able to compute $\mathsf{Ext}(\underline{W}; \underline{S})$. We handle this by using a Markov style argument, as we shall see shortly.

Non-malleability of the underlying NMC: To use the non-malleability of the inner code, $(\mathsf{NMEnc}, \mathsf{NMDec})$, consider the tampering functions f^*, g^* on (L, R) that sample $W, \widetilde{C}$ uniformly and then compute $f^*(L) = f_1(L, W)$ and $g^*(R) = g_1(R, \widetilde{C})$. Let $\underline{S} = \mathsf{NMDec}(f^*(L), g^*(R))$. By the augmented non-malleability of $(\mathsf{NMEnc}, \mathsf{NMDec})$, there exists a simulator NMSim such that

$$\underline{S}, L, S, W, \widetilde{C} \approx \mathsf{Copy}(\mu, \mathsf{NMSim}), S, W, \widetilde{C}\ , \quad (3)$$

where the output of NMSim depends on $W, \widetilde{C}$ and the functions f_1, g_1. Recall (from definition 7) that NMSim is parsed as the joint distribution $(\mathsf{NMSim}_1, L^{\mathsf{Sim}})$. We further denote NMSim_1 by $\underline{S^{\mathsf{Sim}}}$, when NMSim_1 is conditioned to not output same^* or $\perp$. We denote by E_1 the event that

[5] Augmented NMC is a natural extension of NMC (and equivalent of strong extraction property). We require that $\mathsf{NMDec}(\underline{L}, \underline{R})$ is equal or independent of $\mathsf{NMDec}(L, R)$ and revealing L or R doesn't ruin this independence.

$(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_3$ and E_2 the event that $S \neq \underline{S}$. Note that E_1 is the same as E_2. We denote by E_3 the event that $\mathsf{NMSim}_1 \notin \{(\mathsf{same}^*, S)\}$. We first show that $\Pr[E_3]$ is high.

From here, using properties of statistical distance, and expanding out the definition of copy, we show that:

$$(\underline{S}, L, S, W, \widetilde{C})|_{E_2} \approx (\underline{S^{\mathsf{Sim}}}, L^{\mathsf{Sim}}, S, W, \widetilde{C})|_{E_3} \tag{4}$$

We next show that the (average) conditional entropy of W given $\underline{S^{\mathsf{Sim}}}, \underline{L}^{\mathsf{Sim}}, \mathsf{Ext}(\underline{W}; \underline{S})$ is high; here $\underline{L}^{\mathsf{Sim}}$ and $\underline{W}$ are tamperings of L^{Sim}, W and can be computed as their deterministic function. In particular, we have that

$$\widetilde{\mathbf{H}}_\infty \left(W | \underline{S^{\mathsf{Sim}}}, \underline{L}^{\mathsf{Sim}}, \mathsf{Ext}(\underline{W}; \underline{S^{\mathsf{Sim}}}) \right)$$

is high.

Further since S is independent of $\underline{S^{\mathsf{Sim}}}$, we use the strong extractor property of Ext, to show the following:

$$\mathsf{Ext}(W; S) \approx U | S, L^{\mathsf{Sim}}, \widetilde{C}, \underline{L}^{\mathsf{Sim}}, \mathsf{Ext}(\underline{W}; \underline{S^{\mathsf{Sim}}}), \underline{S^{\mathsf{Sim}}} \tag{5}$$

In order to proceed with the proof, we will prove a statement similar to the one above; except that instead of the tampered seed being simulated, we will let it be the tampered seed (in the underlying non-malleable code). (Eventually we would need to replace $\widetilde{C}$ by $C := \mathsf{Ext}(W; S) \oplus m$.) For the remainder of the proof, let $L, W, R, \widetilde{C}$ be distributed as $L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c|_{E_1}$. Also, let $\underline{S}, \underline{L}, \underline{R}, \underline{W}$ be the corresponding tampered random variables. After tampering conditioned on the event that $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_3$. We shorthand the distribution in the LHS and RHS of every equation I by IA and IB, respectively.

$$\mathsf{Ext}(W; \underline{S}), S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}) \approx \mathsf{Ext}(W; \underline{S}), S^{\mathsf{sim}}, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S^{\mathsf{sim}}}) \tag{6}$$

$$\mathcal{U}, S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}) \approx \mathcal{U}, S^{\mathsf{sim}}, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S^{\mathsf{sim}}}) \tag{7}$$

Applying triangle inequality on Inequalities (5), (6), (7), we get that

$$\mathsf{Ext}(W; S) \approx U | S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S} \tag{8}$$

Note that in the equation above, there is no dependence on m on either side as $\widetilde{C}$ is independent of m. Ultimately, we would like to say that the output of the tampering experiment is indistinguishable from the simulated output. We accomplish this in three steps:

1. Adding R. In equation 8, the only information correlated to W and R is $\underline{S}$. Since $\mathsf{Ext}(W; S) \approx \mathcal{U}$ even given $\underline{S}$, we can safely add R to Equation 8.

$$\mathsf{Ext}(W; S) \approx U | S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, R, g_1(R, \widetilde{C}, \widetilde{\sigma}_c) \; .$$

From here, we would ideally like to drop $\widetilde{C}$ and somehow bring back the dependence on m via C. For now, we drop $\widetilde{C}$

$$\mathsf{Ext}(W; S) \approx U | S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, R, \underline{R} \; . \tag{9}$$

The way we'll bring C is to condition $\widetilde{C}$ on being a "cipher of m". For that we first need to prove that $\widetilde{C}$ is independent of W given appropriate auxiliary information.

2. Capturing $\widetilde{C}$'s correlation with W. In this step, we will prove that $\widetilde{C}$ is independent of W given $S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, R, \underline{R}$. We first observe that $\widetilde{C}$ is independent of W given (L, R, S). Now we would like to add the other random variables in the auxiliary information. We use the a Lemma due to Dziembowski and Pietrzak which states that independence in the presence of additional auxiliary information is indeed possible, provided it satisfies a few properties:

- The auxiliary information may be computed in multiple steps.
- Computation in all of the steps can use (L, R, S) and the part of the auxiliary information generated in previous steps.
- Computation in a given step can either depend on $\widetilde{C}$ or W but not both.

By computing auxiliary information in the order $\underline{L}$ followed by $\underline{R}$ followed by $\mathsf{Ext}(\underline{W}; \underline{S})$, we can easily prove that $\widetilde{C}$ is independent of W given $S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, R, \underline{R}$.

3. Conditioning $\widetilde{C}$ appropriately Since W is independent of $\widetilde{C}$ given appropriate auxiallary information, in Equation 9, we can condition $\widetilde{C}$ to either be $m \oplus \mathsf{Ext}(W, S)$ or $m \oplus U$. (Note that the latter is identical to C.) By doing so, Equation 9 will lead to the following $C, S, L, \underline{L}, \mathsf{Ext}(W; S), \underline{S}, R, \underline{R} \approx \mathcal{U}, S, L, \underline{L}, \mathsf{Ext}(W; S), \underline{S}, R, \underline{R}$, where $\underline{R}, \underline{S}$ are appropriately computed.
The desired result follows by observing that the tampered codeword is a function of

$$\underline{L}, \underline{R}, \mathsf{Ext}(\underline{W}; \underline{S}), C, R \; .$$

Putting it Together. So far, we've described the simulator and sketched the proof for showing that the simulated output is indistinguishable from the tampered output in each of the cases. To complete the proof, we need to combine all three cases and, in particular, the probability that the codewords (tampered vs simulated) lie in each of the partitions needs to be analysed.

Candidate Instatiation While we can turn any augmented non-malleable code (or randomness encoder) into a good rate non-malleable code, a very simple result can be obtained using [ADL14]. To encode a message m all we will need is *an affine evasive function* h. It is a function $h : \mathbb{F}_p \to \mathcal{M} \cup \perp$ such that $\Pr(h(aU + b) \neq \perp \mid h(U) = m)$ is negligible for all a, b, m, and $U | h(U) = m$ should be efficiently samplable, the construction of the said function can be found in [ADL14, Agg15]. The encoding procedure is described in Figure 3.

Short and Simple: The Encoding Procedure:

1. Sample s, k_w, k_c, w uniformly at random.
2. Sample x uniformly random, such that $h(x) = s, k_w, k_c$.
3. Sample $\ell, r \in \mathbb{F}_p^n$ uniformly random, such that $\langle \ell, r \rangle = x$.
4. Evaluate $c = \mathsf{Ext}(w; s) \oplus m$.
5. Calculate MACs $\sigma_c = \mathsf{Tag}_{k_c}(c)$ and $\sigma_w = \mathsf{Tag}'_{k_w}(w)$

The final output is: on the left: ℓ, w, σ_w, and on the right: r, c, σ_c.

Figure 3: Simple non-malleable code with a great rate. Here h is an affine evasive function. The decoding procedure is analogous: the decoder inverts Steps 3 and 2, obtains keys k_w, k_c, verifies MACs from the Step 5 and proceeds to obtain the message via the Step 4. If in Step 2 the function h outputs $\perp$, then the decoder aborts and outputs $\perp$.

1.3 Related Work

We now sketch the landscape of this area, and particularly summarize the results on 2-split-state NMCs in Table 1. In [DPW10], in addition to introducing non-malleable codes, Dziembowski *et al.* also introduced a model of tampering called the t-split-state model, where the codeword consists of t independently tamperable states. They give the first NMC constructions in the n-split-state model (where n is the codeword length), with rate ≈ 0.19, and the 2-split-state model (using random oracles). Dziembowski, Kazana and Obremski [DKO13] provided the first construction of 2-split-state NMCs without any assumptions. Their construction enabled encoding of 1-bit messages and used two source extractors. The first NMC in the 2-split-state model for k-bit messages was given by Aggarwal, Dodis and Lovett [ADL14], which used inner product extractors with tools from additive combinatorics. In [CG14a], Cheraghchi and Guruswami brought focus to the rate $= \frac{\mathsf{message\ length}}{\mathsf{codeword\ length}}$ of non-malleable codes. In particular, they showed that the optimal achievable rate for the t-split-state family is $1-1/t$. Note that in the split-state tampering model, having as few states is most desirable, with 2 states being the best achievable. By the above result, the best possible rate for the 2-split-state model is therefore $\frac{1}{2}$. A long series of works[6] [CG14b, CZ14, ADKO15a, CGL16, Li17, Li19, KOS17, KOS18, GMW18, AO20] has made partial progress towards achieving these parameters. We now discuss some of these results. The work of Cheraghchi and Guruswami [CG14b] gave the first optimal rate non-malleable code in the n-split-state model (where n is the codeword length). More importantly, this work introduced non-malleable two-source extractors and demonstrated that these special extractors can be used to generically build 2-split-state NMCs. This connection has led to several fascinating works [CZ14, CGL16, Li17, Li19] striving to improve the rate and number of

[6] Other works have considered non-malleable codes in models other than the 2-split-state model or under computational assumptions [AAG+16, FMNV14, AGM+15, JW15, AKO17], [DNO17, DLSZ20, DKS19, CKR16, ADKO15a, CGM+16, CL17, GMW18, BDSKM18, FHMV17].

Work	Rate
[DPW10]	1/6 (Existential, Random Oracle Model)
[CG14a]	1/2 (Existential, Lower bound)
[DKO13]	$\Omega(1/n)$ (Only for 1-bit messages)
[ADL14, Agg15, AB16]	$\Omega(1/n^{4/5})$
[ADKO15a]	$n^{-\Omega(1)}$
[CGL16]	$n^{-\Omega(1)}$
[Li17]	$\Omega(1/\log(n))$
[Li19]	$\Omega(\log\log(n)/\log(n))$
[Li19]	$\Omega(1)$ (with constant error)
[AO20]	$\approx 1/1,500,000$
Our Result	1/3

Table 1: Prior Work on 2-state NMCs (n is codeword length)

states of non-malleable codes. Most notably, Chattopadhyay and Zuckerman [CZ14] built a 10-state NMC with constant rate, making this the first constant rate construction with sublinear number of states. They achieve their result by first building a non-malleable extractor with 10 sources and then using the connection due to [CG14b]. Aggarwal, Dodis, Kazana and Obremski [ADKO15a] introduced the concept of non-malleable reductions – which would later be used to build constant rate NMCs [AO20]. The work of Kanukurthi, Obbattu and Sekar [KOS17] used seeded extractors to build a compiler that transforms a low rate non-malleable code into one with high rate and, in particular, obtained a rate 1/3, 4−state non-malleable code. This was subsequently improved to three states in the works of Kanukurthi, Obbattu and Sekar [KOS18] as well as Gupta, Maji and Wang [GMW18]. Li [Li19] obtained 2-split-state NMC with rate $O(\frac{\log\log\log(1/\epsilon)}{\log\log(1/\epsilon)})$ (where ϵ is the error). Particularly, this gave a rate of $O(\log\log(n)/\log(n))$, for negligible error $\epsilon = 2^{-\Omega(n)}$, and a constant rate for constant error, making this the first constant rate scheme in the 2−split-state model. The most recent work of Aggarwal and Obremski [AO20] relied on the concept of non-malleable reductions and built the first constant rate 2-split-state NMC with negligible error.

1.4 Organization of the Paper

We describe the preliminary building blocks in Section 2. We then describe our rate boosters for the non-malleable codes in Section 3 and give its application to non-malleable commitments in Section 4. Finally, we describe our rate boosters for the non-malleable randomness extractors in Section 5.

2 Preliminaries

2.1 Notation

We begin by describing some notations which we use. We use $s \in_R S$ to denote that s is sampled uniformly from the set S, and $x \leftarrow X$ to denote that x is sampled from the probability distribution X. The concatenation of two binary strings x and y is denoted by $x\|y$. We denote the length of a binary string x by $|x|$, and the cardinality of any set S by $|S|$. For any set S, U_S denotes the uniform distribution on S, and we use the shorthand U_ℓ to represent $U_{\{0,1\}^\ell}$, for any integer $\ell > 0$. All logarithms are over base 2. The set $X \backslash Y \stackrel{\text{def}}{=} \{x : x \in X, x \notin Y\}$, is the set of elements in X that are not in Y.

For any event E, and any random variable X_1, $X_1|_E$ denotes the distribution of the random variable X_1 conditioned on the event E. For any random variables X_1, X_2 and an event E, $(X_2, X_1|_{E,X_2})$ denotes the distribution where we sample X_2 according to its marginal distribution, and then sample X_1 conditioned on the choice of X_2 and the event E.

2.2 Statistical distance and Entropy

Let X_1, X_2 be two probability distributions over some set S. Their statistical distance is

$$\Delta(X_1; X_2) \stackrel{\text{def}}{=} \max_{T \subseteq S} \{\Pr[X_1 \in T] - \Pr[X_2 \in T]\} = \frac{1}{2} \sum_{s \in S} \left| \Pr_{X_1}[s] - \Pr_{X_2}[s] \right|$$

(they are said to be ε-close if $\Delta(X_1; X_2) \leq \varepsilon$ and denoted by $X_1 \approx_\varepsilon X_2$). We shorthand $\Delta(X_1, Y; X_2, Y)$ by $\Delta(X_1; X_2|Y)$ for any random variables X_1, X_2, Y. The *min-entropy* of a random variable W is $\mathbf{H}_\infty(W) = -\log(\max_w \Pr[W = w])$. For a joint distribution (W, E), the (average) conditional min-entropy of W given E is defined in [DORS08] as

$$\widetilde{\mathbf{H}}_\infty(W \mid E) = -\log(\mathop{\mathbf{E}}_{e \leftarrow E}(\max_w \Pr[W = w|E = e]))$$

(here the expectation is taken over e for which $\Pr[E = e]$ is nonzero). For a random variable W over $\{0,1\}^n$, $W|E$ is said to be an $(n, t)-$source if $\widetilde{\mathbf{H}}_\infty(W|E) \geq t$. We require the following lemma about conditional min-entropy.

Lemma 1. *[DORS08] If B has at most 2^λ possible values, then $\widetilde{\mathbf{H}}_\infty(A \mid B) \geq \mathbf{H}_\infty(A, B) - \lambda \geq \mathbf{H}_\infty(A) - \lambda$, and, more generally, $\widetilde{\mathbf{H}}_\infty(A \mid B, C) \geq \widetilde{\mathbf{H}}_\infty(A, B \mid C) - \lambda \geq \widetilde{\mathbf{H}}_\infty(A \mid C) - \lambda$*

Lemma 2. *For any random variables A, B, if $A \approx_\epsilon B$, then for any (possibly randomized) function f, $f(A) \approx_\epsilon f(B)$*

We require the following lemma from [ADL14], which states that if the pairs of random variables (X_1, X_2) and (Y_1, Y_2) are statistically close, then for any set $\mathcal{A}$, the conditional random variables X_2 conditioned on the event $X_1 \in \mathcal{A}$, i.e., $X_2|_{X_1 \in \mathcal{A}}$, and Y_2 conditioned on the event $Y_1 \in \mathcal{A}$, i.e., $Y_2|_{Y_1 \in \mathcal{A}}$ are also close.

Lemma 3. *[ADL14, Claim 4] Let X_1, X_2, Y_1, Y_2 be random variables such that $(X_1, X_2) \approx_\epsilon (Y_1, Y_2)$. Then, for any non-empty set $\mathcal{A}$, we have:*

$$\Delta(X_2|_{X_1 \in \mathcal{A}}; Y_2|_{Y_1 \in \mathcal{A}}) \leq \frac{2\epsilon}{\Pr[X_1 \in \mathcal{A}]} .$$

We require the following lemmas.

Lemma 4. *[AO20] Let S be some random variable distributed over a set $\mathcal{S}$, and let $\mathcal{S}_1, \ldots, \mathcal{S}_j$ be a partition of $\mathcal{S}$. Let $\phi : \mathcal{S} \to \mathcal{T}$ be some function, and let $D_1, \ldots, D_j$ be some random variables over the set $\mathcal{T}$. Assume that for all $1 \leq i \leq j$,*

$$\Delta\left(\phi(S)|_{S\in\mathcal{S}_i} \; ; \; D_i\right) \leq \varepsilon_i.$$

Then

$$\Delta\left(\phi(S) \; ; \; D\right) \leq \sum \varepsilon_i \Pr[S \in S_i] \; ,$$

for some random variable $D \in \mathcal{T}$ such that for all d $\Pr[D = d] = \sum_i \Pr[S \in \mathcal{S}_i] \cdot \Pr[D_i = d]$.

The following result is from [DP07]. For a proof, see [ADKO15b].

Lemma 5. *Let $A \in \mathcal{A}$ and $B \in \mathcal{B}$, and V_0 be random variables such that A is independent of B given any fixing of the random variable V_0. Let $V_1, V_2, \ldots$ be random variables defined as functions of A, B satisfying the following property. For all $i \in \mathbb{N}$, if i is even then $V_i = \phi_i(V_0, V_1, \ldots, V_{i-1}, A)$ and if i is odd, then $V_i = \phi_i(V_0, V_1, \ldots, V_{i-1}, B)$ for some function ϕ_i. Then for all i, A is independent of B given any fixing of $V_0, V_1, \ldots, V_i$.*

Further, we require the following simple lemma about the statistical distance between conditional distributions, given the distance between a pair of joint distributions.

Lemma 6. *For any random variables X, Y and Z and uniform random variable U, with X, Y, U defined over some binary space $\mathcal{X}$, such that Y is independent of X conditioned on Z and also independent of U conditioned on Z, if $(X, Z) \approx_\varepsilon (U, Z)$, then it implies that $(Y, Z)|(X \oplus Y = x) \approx_\varepsilon (U, Z)|(U \oplus Y = x)$, for any $x \in \mathcal{X}$.*

Proof. Assume to the contrary that there exists an unbounded adversary, $\mathcal{D}$, who can distinguish between $(Y, Z)|(X \oplus Y = x)$ and $(U, Z)|(U \oplus Y = x)$ for some $x \in \mathcal{X}$, with advantage $> \varepsilon$. Then, we claim that we can build an adversary distinguishing (X, Z) and (U, Z) with the same advantage. The reduction is simple: given (a, c) from (X, Z) or (U, Z), we sample $b \leftarrow Y|a \oplus Y = x$. Given the independence of Y from both $X|Z$ and $U|Z$, (b, c) is correctly simulated to be either $(Y, Z)|(X \oplus Y = x)$ (if (a, c) was from (X, Z)) or $(U, Z)|(U \oplus Y = x)$ (if (a, c) was from (U, Z)), for $\mathcal{D}$. Hence, the lemma is proved. □

2.3 Randomness Extractors

Definition 1 (Seeded Extractors)**.** *We say that a polynomial time probabilistic function* $\mathsf{Ext} : \{0,1\}^n \times \{0,1\}^d \to \{0,1\}^\ell$*, using d bits of randomness, is an $(n, k, d, \ell, \varepsilon)$-*strong seeded extractor *if for all random variables $W \in \{0,1\}^n$, E correlated with W, such that $\mathbf{H}_\infty(W|E) \geq k$, and X independent of W, E and uniform in $\{0,1\}^d$, we have $(\mathsf{Ext}(W; X), X, E) \approx_\varepsilon (U_\ell, X, E)$, where U is uniform in $\{0,1\}^\ell$ and independent of X, E.*

We require the following seeded extractor construction from [GUV07].

Lemma 7. *[GUV07] For every constant $\nu > 0$, all integers $n \geq k$ and all $\varepsilon \geq 0$, there is an explicit (efficient) $(n, k, d, \ell, \varepsilon)-$strong seeded extractor with $\ell = k - \mathcal{O}\left(\log n + \log \frac{1}{\varepsilon}\right)$ and $d = \mathcal{O}\left(\log n + \log \frac{1}{\varepsilon}\right)$.*

2.4 One-Time Message Authentication Codes

Our construction also requires the use of information theoretic one-time message authentication codes, which give a guarantee that, given a message-tag pair, (m, t), where t is generated using a tag generation algorithm, using a random authentication key, the probability with which an (unbounded) adversary can come up with a valid message-tag pair (m', t'), for $m' \neq m$, is negligible.

Definition 2. *A family of functions* $\{\mathsf{Tag}_{k_a} : \{0,1\}^\gamma \to \{0,1\}^\delta, \mathsf{Vrfy}_{k_a} : \{0,1\}^\gamma \times \{0,1\}^\delta \to \{0,1\}\}_{k_a \in \{0,1\}^\tau}$ *is said to be a* $\mu-$*secure one time message authentication code if*

1. *For* $k_a \in_R \{0,1\}^\tau$, $\forall\, m \in \{0,1\}^\gamma$, $\Pr[\mathsf{Vrfy}_{k_a}(m, \mathsf{Tag}_{k_a}(m)) = 1] = 1$,
   where for any (m,t), $\mathsf{Vrfy}_{k_a}(m,t) \stackrel{def}{=} \begin{cases} 1 \text{ if } \mathsf{Tag}_{k_a}(m) = t \\ 0 \text{ otherwise} \end{cases}$
2. *For any* $m \neq m', t, t'$, $\Pr\limits_{k_a}[\mathsf{Tag}_{k_a}(m) = t | \mathsf{Tag}_{k_a}(m') = t'] \leq \mu$, *where* $k_a \in_R \{0,1\}^\tau$.

For simplicity, we consider Tag to be a deterministic function.
The following lemma guarantees that there exists efficient one-time message authentication codes where the key and tag can be much shorter than the message to be authenticated.

Lemma 8. *[JKS93] For any* $\gamma, \varepsilon > 0$ *there is an efficient* $\varepsilon-$*secure one time* MAC *with* $\delta \leq (\log(\gamma) + \log(\frac{1}{\varepsilon}))$, $\tau \leq 2\delta$, *where* τ, γ, δ *are key, message, tag length respectively.*

We refer the reader to [DKK+12] for a construction satisfying these parameters.

2.5 Non-malleable Codes

Non-malleable codes (NMCs) were introduced in [DPW10]. We now state the equivalent definition of non-malleable codes given in [CG14b].

Definition 3. *A (possibly randomised) function pair* $(\mathsf{Enc} : \mathcal{M} \to \mathcal{C}, \mathsf{Dec} : \mathcal{C} \to \mathcal{M} \cup \{\perp\})$ *is said to be a* ***coding scheme*** *from* $\mathcal{M}$ *to* $\mathcal{C}$ *if* $\forall m \in \mathcal{M}$, $\Pr[\mathsf{Dec}(\mathsf{Enc}(m)) = m] = 1$ *(the probability is over the randomness of* $\mathsf{Enc}, \mathsf{Dec}$*). The rate of the coding scheme is given by* $\frac{\log|\mathcal{M}|}{\log|\mathcal{C}|}$.

Definition 4. *A coding scheme* $(\mathsf{Enc}, \mathsf{Dec})$ *from* $\mathcal{M}$ *to* $\mathcal{C}$, *is said to be* ϵ- ***non-malleable*** *with respect to a tampering function family* $\mathcal{F} \subseteq \{f : \mathcal{C} \to \mathcal{C}\}$ *if* $\forall\ f \in \mathcal{F}$, *there exists a distribution* NMSim *(specific to* f*) over* $\mathcal{M} \cup \{\mathsf{same}^*, \perp\}$ *such that* $\forall\ m \in \mathcal{M}$

$$\mathsf{Tamper}_f^m \approx_\epsilon \mathsf{Copy}(m, \mathsf{NMSim})$$

where Tamper_f^m *denotes the distribution* $\mathsf{Dec}(f(\mathsf{Enc}(m)))$ *and* $\mathsf{Copy}(m, \mathsf{NMSim})$ *is defined as*

$$\tilde{m} \leftarrow Sim_f$$

$$\mathsf{Copy}(m, \mathsf{NMSim}) = \begin{cases} m \ if\ \tilde{m} = \mathsf{same}^* \\ \tilde{m} \text{ otherwise} \end{cases}$$

NMSim *should be efficiently samplable given oracle access to* $f(.)$.

Split-state Tampering Family. Specifically, we consider NMCs with respect to the two split-state tampering family defined below, for message space $\{0,1\}^\alpha$ and codeword space $\{0,1\}^\beta \times \{0,1\}^\beta$, comprising two states:

$$\mathcal{F}_{\mathsf{split}} \stackrel{\text{def}}{=} \{(f,g) : f,g : \{0,1\}^\beta \to \{0,1\}^\beta\}$$

Our construction requires NMCs with respect to the split-state tampering family satisfying an additional property called "augmented security", which was introduced in [AAG+16], and guarantees that in addition to the tampered message, one of the states, say L, of the codeword is also simulatable independent of the message. We formally define NMCs against the split-state model, with augmented security below.

Definition 5 (Augmented Non-malleable Codes)**.** *A coding scheme* $(\mathsf{Enc}, \mathsf{Dec})$ *from* $\{0,1\}^\alpha$ *to* $\{0,1\}^\beta \times \{0,1\}^\beta$ *is called an* ϵ*-augmented non-malleable code with respect to* $\mathcal{F}_{\mathsf{split}}$*, if the following holds. For any (possibly randomized)* $(f,g) \in \mathcal{F}_{\mathsf{split}}$*, let* $\mathsf{Tamper}^m_{f,g}$ *denote the distribution* $\mathsf{Dec}(f(L), g(R))$*, for* $(L,R) \leftarrow \mathsf{Enc}(m)$*. There exists a simulator that, given oracle access to* $(f,g)(.)$*, outputs* $\mathsf{NMSim} = (\mathsf{NMSim}_1, \mathsf{NMSim}_2)$ *over* $(\{0,1\}^\alpha \cup \{same^*, \perp\}) \times \{0,1\}^\beta$ *such that, for all* $m \in \{0,1\}^\alpha$

$$\mathsf{Tamper}^m_{f,g}, L \approx_\epsilon \mathsf{Copy}(m, \mathsf{NMSim});,$$

where $(\mathsf{NMSim}_1, \mathsf{NMSim}_2) = (\tilde{m}, L^{sim}) \leftarrow \mathsf{NMSim}$*, and* $\mathsf{Copy}(m, \mathsf{NMSim})$ *is defined as*

$$\mathsf{Copy}(m, \mathsf{NMSim}) \stackrel{def}{\equiv} \begin{cases} (m, L^{sim}) \text{ if } \tilde{m} = \mathsf{same}^* \\ (\tilde{m}, L^{sim}) \text{ otherwise} \end{cases}$$

We call the above code as left-augmented (NMSim *outputs the left state), and say that the code is right-augmented when the simulator outputs the right state instead.*

We also require the following secret sharing property of non-malleable codes in the 2-split-state model $\mathcal{F}_{\mathsf{split}}$. It states that any 2-split-state non-malleable code is a 2-out-of-2 secret sharing scheme.

Lemma 9. *[ADKO15a] Let* $\mathsf{Enc} : \{0,1\}^\alpha \to \{0,1\}^\beta \times \{0,1\}^\beta$ *and* $\mathsf{Dec} : \{0,1\}^\beta \times \{0,1\}^\beta \to \{0,1\}^\alpha \cup \{\perp\}$ *be an* ϵ*-non-malleable code in the* 2*-split-state model for some* $\epsilon < 1/2$*. For any pair of messages* $m_0, m_1 \in \{0,1\}^\alpha$*,* $R^{m_0} \approx_{2\epsilon} R^{m_1}$*, where* $(L^{m_0}, R^{m_0}) \leftarrow \mathsf{Enc}(m_0)$ *and* $(L^{m_1}, R^{m_1}) \leftarrow \mathsf{Enc}(m_1)$*.*

We instantiate our NMC with two constructions, one due to [ADL14, Agg15, AB16] and another due to [Li19], which are both stated below in Theorems 1 and 2, respectively. While the former instantiation gives us a very simple non-malleable code, as explained in the introduction, the latter instantiation gives us a better error for the same rate.

Theorem 1. *[ADL14, Agg15, AB16] There exists and efficient construction of an* ε*-non-malleable code in the split state model with message space* $\{0,1\}^\kappa$ *and codeword space* $\mathbb{F}_p^q \times \mathbb{F}_p^q$*, for a prime* $p \leq 2^{O(\kappa)}$*,* $q = O(\kappa^4)$ *and* $\varepsilon = 2^{-\Omega(\kappa)}$*.*

Theorem 2. *[Li19, Theorem 1.15] There are constants* $0 < c_1, c_2 < 1$ *such that for any* $\beta \in \mathbb{N}$ *and* $2^{-\frac{c_2\beta}{\log\beta}} \leq \epsilon \leq c_1$*, there exists an explicit non-malleable code in the* 2*-split-state model with block length* 2β*, rate* $\Omega(\frac{\log\log\log(1/\epsilon)}{\log\log(1/\epsilon)})$ *and error* ϵ*.*

The NMC in Theorem 1 is shown to be augmented secure in [AAG+16]. Further, the NMC in Theorem 2 relies on building a two source non-malleable extractor and then using the [CG14b] compiler to get a 2-split-state NMC. Hence, we can show that the NMC in Theorem 2 is in fact an augmented NMC using the following two observations: 1) the two source non-malleable extractor constructed in [Li19] is in fact, a strong two source non-malleable extractor (which allows leaking one complete source); 2) as proved in [KOS18, Proposition 2], the [CG14b] compiler gives an augmented NMC, when a strong two-source non-malleable extractor is used.

3 A Rate Booster for Non-malleable Codes

In this section, we prove the following result.

Theorem 3. *Let* $(\mathsf{NMEnc} : \{0,1\}^{\kappa} \to \{0,1\}^n \times \{0,1\}^n,\ \mathsf{NMDec} : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^{\kappa})$ *be an* ε*-augmented non-malleable code in the split-state model. Then, for any* $\kappa^* \in \omega(n)$*, there exists an efficient* $O(\sqrt{\varepsilon} + 2^{-\Omega(\kappa/\log \kappa)})$*-augmented non-malleable code (specifically, right-augmented, i.e., the simulator outputs the right state along with the tampered message) from* κ^**-bit messages to at most* $(3 + o(1))\kappa^*$*-bit codewords.*

In Section 3.1, we will give our construction, and its security is proved in Section 3.2. Using $(\mathsf{NMEnc}, \mathsf{NMDec})$ from Theorem 1, we get the following instantiation, which is conceptually very simple, as shown in Figure 3 in the introduction.

Corollary 1. *For any integer* $\kappa^* > 0$*,* $\varepsilon^* \in 2^{-\Omega((\kappa^*)^{1/5}/\operatorname{polylog}(\kappa^*))}$*, there is an efficient* ε^**-right-augmented non-malleable code from* κ^**-bit messages to* $(3 + o(1))\kappa^*$*-bit codewords in the split-state model.*

Further, using $(\mathsf{NMEnc}, \mathsf{NMDec})$ from Theorem 2, and setting $\kappa^* = n \log n$ we can get the following instantiation, with a better error at the expense of being more complicated.

Corollary 2. *For any integer* $\kappa^* > 0$*,* $\varepsilon^* \in 2^{-\Omega(\kappa^*/\log^3 \kappa^*)}$*, there is an efficient* ε^**-right-augmented non-malleable code from* κ^**-bit messages to* $(3 + o(1))\kappa^*$*-bit codewords in the split-state model.*

3.1 Our construction

Building Blocks Let $N = 2\kappa^* + 8n$. Our construction, shown in Figure 4 requires the following building blocks.

- Let Ext be the $(N,k,d,\ \kappa^*,\varepsilon_2)$-strong seeded extractor from Lemma 7 with $k = \kappa^* + 4n$, and $\varepsilon_2 = 2^{-\Omega(\kappa/\log \kappa)}$.
- Let $(\mathsf{Tag}, \mathsf{Vrfy})$ be the one time ε_3-message authentication codes from Lemma 8 with key length $\tau = \Theta(\kappa)$, messages of size at most N, tag length $t = \Theta(\kappa)$, and $\varepsilon_3 = 2^{-\Omega(\kappa)}$.

We now describe our construction below. The encoder chooses a source w and a seed s corresponding to a seeded extractor Ext and then computes $c := m \oplus \mathsf{Ext}(w; s)$, where $\oplus$ is the bitwise XOR. Further, it authenticates the source w and the ciphertext c to get the tags σ_w and σ_c respectively, generates the non-malleable encoding (using the underlying NMC) of the seed and the authentication keys to get (ℓ, r) and finally outputs (ℓ, w, σ_w) as one state and (r, c, σ_c) as the other.

$\mathsf{Enc}(m)$:

- $s \in_R \{0,1\}^d$, $k_w, k_c \in_R \{0,1\}^{\tau}$, $w \in_R \{0,1\}^N$
- $c = m \oplus \mathsf{Ext}(w; s)$.
- $\sigma_w = \mathsf{Tag}_{k_w}(w)$ and $\sigma_c = \mathsf{Tag}_{k_c}(c)$
- $(\ell, r) \leftarrow \mathsf{NMEnc}(\mu)$ where $\mu = (s, k_w, k_c)$
- Output $(\ell, w, \sigma_w), (r, c, \sigma_c)$

$\mathsf{Dec}((\ell, w, \sigma_w), (r, c, \sigma_c))$:

- $(s, k_w, k_c) = \mathsf{NMDec}(\ell, r)$.
- If $\mathsf{Vrfy}_{k_w}(w, \sigma_w) = 1$ and $\mathsf{Vrfy}_{k_c}(c, \sigma_c) = 1$, Output $m = \mathsf{Ext}(w; s) \oplus c$.

  Else, Output $\perp$.

Figure 4: Our NMC Construction

3.2 Security Proof

The correctness of the construction is immediate by definition. We now prove the desired non-malleability.

Fix any $m \in \{0,1\}^{\kappa^*}$, tampering functions $(f,g) \in \mathcal{F}_{\mathsf{split}}$. Throughout this proof, we use the notation $\underline{X}$ to denote any part X of the message/codeword after tampering. Let $f = (f_1, f_2, f_3)$ and $g = (g_1, g_2, g_3)$, where

$$f_1 : \{0,1\}^{n+N+t} \to \{0,1\}^n,\ f_2 : \{0,1\}^{n+N+t} \to \{0,1\}^N,\ f_3 : \{0,1\}^{n+N+t} \to \{0,1\}^t\ ,$$

and

$$g_1 : \{0,1\}^{n+\kappa^*+t} \to \{0,1\}^n,\ g_2 : \{0,1\}^{n+\kappa^*+t} \to \{0,1\}^{\kappa^*},\ g_3 : \{0,1\}^{n+\kappa^*+t} \to \{0,1\}^t\ .$$

Let $W, S, K_w, K_c, C, \sigma_w, \sigma_c, L, R$ be the distributions of $w, s, k_w, k_c, c, \sigma_w, \sigma_c, \ell, r$ [7] sampled/computed in $\mathsf{Enc}(m)$. Now, apply the tampering to get $(\underline{L}, \underline{W}, \underline{\sigma_w}) = f(L, W, \sigma_w)$, $(\underline{R}, \underline{C}, \underline{\sigma_c}) = g(R, C, \sigma_c)$, decode to get $\underline{\mu} = \mathsf{NMDec}(\underline{L}, \underline{R})$ and $\underline{M} = \mathsf{Dec}((\underline{L}, \underline{W}, \underline{\sigma_w}), (\underline{R}, \underline{C}, \underline{\sigma_c}))$. Further, note that the distribution $\underline{M}$ is the distribution $\mathsf{Tamper}^m_{f,g}$ itself.

Our proof proceeds by partitioning the sample space of the codeword, $\{0,1\}^n \times \{0,1\}^N \times \{0,1\}^t \times \{0,1\}^n \times \{0,1\}^{\kappa^*} \times \{0,1\}^t$ and proving non-malleability in each of the partitions. Consider the following three partitions.

$$\mathcal{P}_1 = \{(\ell, w, \sigma_w, r, c, \sigma_c)\ :\ \ (f_2(\ell, w, \sigma_w), f_3(\ell, w, \sigma_w), g_2(r, c, \sigma_c), g_3(\ell, c, \sigma_c)) = (w, \sigma_w, c, \sigma_c),\ \mathsf{NMDec}(f_1(\ell, w, \sigma_w), g_1(r, c, \sigma_c)) = \mathsf{NMDec}(\ell, r)\}\ ,$$

$$\mathcal{P}_2 = \{(\ell, w, \sigma_w, r, c, \sigma_c)\ :\ \ (f_2(\ell, w, \sigma_w), f_3(\ell, w, \sigma_w), g_2(r, c, \sigma_c), g_3(\ell, c, \sigma_c)) \neq (w, \sigma_w, c, \sigma_c),\ \mathsf{NMDec}(f_1(\ell, w, \sigma_w), g_1(r, c, \sigma_c)) = \mathsf{NMDec}(\ell, r)\}\ ,$$

and

$$\mathcal{P}_3 := \{(\ell, w, \sigma_w, r, c, \sigma_c)\ :\ \mathsf{NMDec}(f_1(\ell, w, \sigma_w), g_1(r, c, \sigma_c)) \neq \mathsf{NMDec}(\ell, r)\}\ ,$$

The event that the codeword is in partition $\mathcal{P}_1$ corresponds to the event when, even after tampering, the decoding of the inner codeword (L, R) remains unchanged, and also W, σ_w, C, σ_c remain unchanged, and hence the final tampered codeword must decode to the original message. The event that the codeword is in partition $\mathcal{P}_2$ corresponds to the event that the decoding of the inner code remains unchanged after tampering, while one of the pairs (W, σ_w) or (C, σ_c) are changed. We will show that if this event occurs then, using the security of MACs, the tampering is detected with overwhelming probability. The event that the codeword is in partition $\mathcal{P}_3$ corresponds to the event that the inner code is non-trivially tampered and does not decode to $\mathsf{Dec}(L, R)$. In this case, we will show that the tampered codeword (and hence its decoding) is independent of the original message m.

We begin by showing that the event that the codeword $(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_i$ for some $i \in \{1, 2, 3\}$ is independent of the message m even given R, C, σ_c.

Let $\widetilde{C}$ be uniform in $\{0,1\}^{\kappa^*}$ and independent of S, K_w, K_c, W, σ_w, and hence also of L, R. Let $\widetilde{\sigma}_c = \mathsf{Tag}_{K_c}(\widetilde{C})$. Let $b_1, b_2, \widetilde{b}_2$ be boolean random variables defined as follows. The random variable b_1 indicates whether W, σ_w changes after tampering, i.e., $b_1 = 1$ if and only if

[7] We slightly abuse notation and let σ_w and σ_c denote both the distribution of the authentication tags σ_w and σ_c as well as samples from those distributions.

$(f_2(L, W, \sigma_w), f_3(L, W, \sigma_w)) = (W, \sigma_w)$. Similarly, the random variable b_2 (respectively, $\widetilde{b}_2$) indicates whether C, σ_c (respectively, $\widetilde{C}, \widetilde{\sigma}_c$) changes after tampering, i.e., $b_2 = 1$ (respectively, $\widetilde{b}_2 = 1$) if and only if $(g_2(R, C, \sigma_c),\ g_3(R, C, \sigma_c)) = (C, \sigma_c)$ (respectively, $(g_2(R, \widetilde{C}, \widetilde{\sigma}_c), g_3(R, \widetilde{C}, \widetilde{\sigma}_c)) = (\widetilde{C}, \widetilde{\sigma}_c)$). Notice that for any i, whether the event that $(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_i$ (respectively, $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_i$) occurs is determined by the random variables $L, R, f_1(L, W, \sigma_w), g_1(R, C, \sigma_c)$, b_1, and b_2 (respectively, $L, R, f_1(L, W, \sigma_w),\ g_1(R, \widetilde{C}, \widetilde{\sigma}_c),\ b_1$, and $\widetilde{b}_2$).

Lemma 10.

$$\Delta\left(C, \sigma_c; \widetilde{C}, \widetilde{\sigma}_c \mid L, R, f_1(L, W, \sigma_w), b_1\right) \leq 4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)} .$$

Proof. By Lemma 9, we have that one of the shares of a split-state non-malleable code is 2ε-independent of the message. In other words, there is a random variable L^* independent of S, K_w, K_c such that

$$L, S, K_w, K_c \approx_{2\varepsilon} L^*, S, K_w, K_c ,$$

Introducing fresh random variable W that is sampled independent of L, S, K_w, K_c, and observing that σ_w is a deterministic function of K_w and W, we have that

$$L, S, K_w, K_c, W, \sigma_w \approx_{2\varepsilon} L^*, S, K_w, K_c, W, \sigma_w . \tag{10}$$

Let b_1^* be a boolean random variable that is 1 if and only if $(f_2(L^*, W, \sigma_w), f_3(L^*, W, \sigma_w)) = (W, \sigma_w)$. By Lemma 1, the average min-entropy of W given $b_1^*,\ f_1(L^*, W, \sigma_w)$ is at least $N - n - 1$, which is at least k. Thus, by Lemma 7, we have that

$$\Delta(\mathsf{Ext}(W; S)\ ;\ U \mid b_1^*, f_1(L^*, W, \sigma_w), L^*, K_w, K_c, S) \leq \varepsilon_2 , \tag{11}$$

where U is uniform in $\{0,1\}^{\kappa^*}$ and independent of all other random variables.
Applying Lemma 2 to Equation 10 twice, we get

$$\Delta(b_1^*, f_1(L^*, W, \sigma_w), L^*\ ;\ b_1, f_1(L, W, \sigma_w), L \mid \mathsf{Ext}(W; S), K_w, K_c, S) \leq 2\varepsilon , \tag{12}$$

and

$$\Delta(b_1^*, f_1(L^*, W, \sigma_w), L^*\ ;\ b_1, f_1(L, W, \sigma_w), L \mid U, K_w, K_c, S) \leq 2\varepsilon . \tag{13}$$

By triangle inequality on inequalities 11, 12, 13, we get that

$$\Delta(\mathsf{Ext}(W; S)\ ;\ U \mid b_1, f_1(L, W, \sigma_w), L, K_w, K_c, S) \leq \varepsilon_2 + 4\varepsilon .$$

Notice that the distribution R is independent of W and is only correlated to (L, K_w, K_c, S) (as $(L, R) \equiv \mathsf{NMEnc}(K_w, K_c, S)$). Hence by applying Lemma 2, we can include R to get that

$$\Delta(\mathsf{Ext}(W; S)\ ;\ U \mid b_1, f_1(L, W, \sigma_w), L, R, K_w, K_c, S) \leq \varepsilon_2 + 4\varepsilon .$$

Finally, observing that given $(b_1, f_1(L, W, \sigma_w), L, R, K_w, K_c, S)$, the random variable $\mathsf{Ext}(W; S) \oplus m$ is distributed as C, and $U \oplus m$ is distributed as $\widetilde{C}$, we get the desired result by another application of Lemma 2. □

The following lemma bounds the probability that the codeword is in $\mathcal{P}_2$ and doesn't decode to $\perp$.

Lemma 11.

$$\Pr[\mathsf{Tamper}_{f,g}^m \neq \perp \ \wedge\ (L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_2] \leq 4\varepsilon + 2^{-\Omega(\kappa)} .$$

Proof. Since the codeword is in $\mathcal{P}_2$, we have that $\mathsf{Dec}(L,R) = \mathsf{Dec}(\underline{L},\underline{R})$, and hence K_w, K_c remain unchanged after tampering. Moreover, one of the pairs (W,σ_w) or (C,σ_c) has been tampered with. Assume without loss of generality that (W,σ_w) has been changed after tampering. By the security of MAC, this tampering will be detected by our decoding algorithm as long as the tampering is independent of K_w. We argue using the secret sharing property of non-malleable codes that this is indeed the case since L is almost independent of K_w.

By Lemma 9, we have that one of the shares of a split-state non-malleable code is 2ε-independent of the message. In other words, there is a random variable L^* independent of S, K_w, K_c such that

$$L, S, K_w, K_c \approx_{2\varepsilon} L^*, S, K_w, K_c \ ,$$

Introducing fresh random variable W that is sampled independent of L, S, K_w, K_c, and observing that σ_w is a deterministic function of K_w and W, we have that

$$L, S, K_w, K_c, W, \sigma_w \approx_{2\varepsilon} L^*, S, K_w, K_c, W, \sigma_w \ .$$

Also, by definition of one-time message authentication codes, we have that

$$\Pr[\mathsf{Vrfy}_{K_w}(f_2(L^*,W,\sigma_w), f_3(L^*,W,\sigma_w)) = 1 \ \wedge \ (f_2(L^*,W,\sigma_w), f_3(L^*,W,\sigma_w)) \neq (W,\sigma_w)] = 2^{-\Omega(\kappa)} \ .$$

By triangle inequality, we get

$$\Pr[\mathsf{Vrfy}_{K_w}(f_2(L,W,\sigma_w), f_3(L,W,\sigma_w)) = 1 \ \wedge \ (f_2(L,W,\sigma_w), f_3(L,W,\sigma_w)) \neq (W,\sigma_w)] \leq 2^{-\Omega(\kappa)} + 2\varepsilon \ .$$

Similarly, there exists R^* independent of S, K_w, K_c such that

$$R, S, K_w, K_c \approx_{2\varepsilon} R^*, S, K_w, K_c \ ,$$

and observing that C is independent of R, K_w, K_c given S, we have that

$$R, S, K_w, K_c, C, \sigma_c \approx_{2\varepsilon} R^*, S, K_w, K_c, C, \sigma_c \ .$$

This implies, via an argument similar as above, that

$$\Pr[\mathsf{Vrfy}_{K_c}(g_2(R,C,\sigma_c), f_3(R,C,\sigma_c)) \ \wedge \ (g_2(R,C,\sigma_c), g_3(R,C,\sigma_c) \neq (C,\sigma_c)] \leq 2^{-\Omega(\kappa)} + 2\varepsilon \ .$$

The desired result then follows from a simple union bound by observing that if $\mathsf{Tamper}^m_{f,g} \neq \perp$, and the codeword is in $\mathcal{P}_2$, i.e., K_w, K_c remain unchanged after tampering, while W, σ_w, C, σ_c are changed after tampering, then we must have that

- $\mathsf{Vrfy}_{K_w}(f_2(L,W,\sigma_w), f_3(L,W,\sigma_w)) = 1$
- $\mathsf{Vrfy}_{K_c}(g_2(R,C,\sigma_c), f_3(R,C,\sigma_c)) = 1$,
- At least one of $(f_2(L,W,\sigma_w), f_3(L,W,\sigma_w)) \neq (W,\sigma_w)$ or $(g_2(R,C,\sigma_c), g_3(R,C,\sigma_c)) \neq (C,\sigma_c)$.

□

Lemma 12. *For any fixed k_w, k_c, and any $\alpha > 2\varepsilon + 2^{-d}$, if $\Pr[(L,W,\sigma_w,R,\widetilde{C},\widetilde{\sigma}_c) \in \mathcal{P}_3] \geq \alpha$, then the statistical distance between*

$$\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_3}$$

and

$$\mathsf{Dec}(f(L,W,\sigma_w), g(R,\widetilde{C},\widetilde{\sigma}_c)), R, \widetilde{C}, \widetilde{\sigma}_c|_{(L,W,\sigma_w,R,\widetilde{C},\widetilde{\sigma}_c)\in\mathcal{P}_3}$$

is at most $2^{-\Omega(\kappa/\log\kappa)} + \frac{4\varepsilon}{\alpha}$.

Proof. In this lemma, we wish to argue that the decoding of the tampered codeword is independent of the message m. We begin by using that since the codeword is in $\mathcal{P}_3$, the output of the tampered codeword for the inner code $\mathsf{NMEnc}(S, k_w, k_c)$ is independent of S which is the seed of the strong extractor.

We will use the non-malleability of the inner code, $(\mathsf{NMEnc}, \mathsf{NMDec})$. Consider the tampering functions f^*, g^* on (L, R) that sample $W, \widetilde{C}$ uniformly and then compute $f^*(L) = f_1(L, W, \mathsf{Tag}_{k_w}(W))$ and $g^*(R) = g_1(R, \widetilde{C}, \mathsf{Tag}_{k_c}(\widetilde{C}))$. Let $\underline{S}, \underline{K_w}, \underline{K_c} = \mathsf{NMDec}(f^*(L), g^*(R))$.[8] By the ε-augmented non-malleability of $(\mathsf{NMEnc}, \mathsf{NMDec})$, there exists a simulator NMSim such that

$$\underline{S}, \underline{K_w}, \underline{K_c}, L, S, W, \widetilde{C} \approx_\varepsilon \mathsf{Copy}(\mu, \mathsf{NMSim}), S, W, \widetilde{C} \ , \tag{14}$$

where the output of NMSim depends on $W, \widetilde{C}$ and the functions f_1, g_1. Recall (from definition 7) that NMSim is parsed as the joint distribution $(\mathsf{NMSim}_1, L^{\mathsf{Sim}})$. We further parse NMSim_1 as $(\underline{S^{\mathsf{Sim}}}, \underline{K_w^{\mathsf{Sim}}}, \underline{K_c^{\mathsf{Sim}}})$, when NMSim_1 is conditioned to not output same^* or $\perp$. If $\mathsf{NMSim}_1 = \perp$, set $(\underline{S^{\mathsf{Sim}}}, \underline{K_w^{\mathsf{Sim}}}, \underline{K_c^{\mathsf{Sim}}}) = (\perp, \perp, \perp)$. The event that $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_3$ is the same as $(\underline{S}, \underline{K_w}, \underline{K_c}) \neq (\underline{S, k_w, k_c})$. By inequality (14) and the hypothesis of the lemma, we have that $\Pr[\mathsf{NMSim}_1 \notin \{\mathsf{same}^*, (S, k_w, k_c)\}] \geq \alpha - \varepsilon$.
By Lemma 3, we have that

$$(\underline{S}, \underline{K_w}, \underline{K_c}, L, S, W, \widetilde{C})|_{(\underline{S}, \underline{K_w}, \underline{K_c}) \neq (S, k_w, k_c)} \approx_{\frac{2\varepsilon}{\alpha}} \mathsf{Copy}(\mu, \mathsf{NMSim}), S, W, \widetilde{C}|_{\mathsf{NMSim}_1 \notin \{\mathsf{same}^*, (S, k_w, k_c)\}}$$

$$(\underline{S}, \underline{K_w}, \underline{K_c}, L, S, W, \widetilde{C})|_{(\underline{S}, \underline{K_w}, \underline{K_c}) \neq (S, k_w, k_c)} \approx_{\frac{2\varepsilon}{\alpha}} (\underline{S^{\mathsf{Sim}}}, \underline{K_w^{\mathsf{Sim}}}, \underline{K_c^{\mathsf{Sim}}}, L^{\mathsf{Sim}}, S, W, \widetilde{C})|_{\mathsf{NMSim}_1 \notin \{\mathsf{same}^*, (S, k_w, k_c)\}} \tag{15}$$

We have that for any w,

$$\Pr[W = w | \mathsf{NMSim}_1 \notin \{\mathsf{same}^*, (S, k_w, k_c)\}] \leq \frac{\Pr[W = w]}{\Pr[\mathsf{NMSim}_1 \notin \{\mathsf{same}^*, (S, k_w, k_c)\}]} \leq \frac{2^{-N}}{\alpha - \varepsilon} \leq 2^{d-N} \ ,$$

where we use that $\alpha > 2\varepsilon + 2^{-d}$. Thus $\mathbf{H}_\infty(W | \mathsf{NMSim}_1 \notin \{\mathsf{same}^*, (S, k_w, k_c)\}) \geq N - d$.
For the remainder of the proof, we assume $\mathsf{NMSim}_1 \notin \{\mathsf{same}^*, (S, k_w, k_c)\}$ (or, when appropriate, $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_3$) but don't state them explicitly.
Let $\mathsf{Vrfy}_\ell^{\mathsf{Sim}} := \mathsf{Vrfy}_{\underline{K_w^{\mathsf{Sim}}}}(f_2(L^{\mathsf{Sim}}, W, \sigma_w), f_3(L^{\mathsf{Sim}}, W, \sigma_w))$, where $\sigma_w = \mathsf{Tag}_{k_w}(W)$. By Lemma 1, we have that

$$\widetilde{\mathbf{H}}_\infty \left(W | \underline{S^{\mathsf{Sim}}}, \underline{K_w^{\mathsf{Sim}}}, \underline{K_c^{\mathsf{Sim}}}, \mathsf{Vrfy}_\ell^{\mathsf{Sim}}, f_1(L^{\mathsf{Sim}}, W, \sigma_w), \mathsf{Ext}(f_2(L^{\mathsf{Sim}}, W, \sigma_w); \underline{S^{\mathsf{Sim}}}) \right)$$

is at least $N - d - \kappa^* - \kappa - n - 1 \geq \kappa^* + 4n$. Thus, by the strong extractor property of Ext,

$$\mathsf{Ext}(W; S) \approx_{\varepsilon_2} U | S, L^{\mathsf{Sim}}, \widetilde{C}, f_1(L^{\mathsf{Sim}}, W, \sigma_w), \mathsf{Ext}(f_2(L^{\mathsf{Sim}}, W, \sigma_w); \underline{S^{\mathsf{Sim}}}), \underline{S^{\mathsf{Sim}}}, \underline{K_w^{\mathsf{Sim}}}, \underline{K_c^{\mathsf{Sim}}}, \mathsf{Vrfy}_\ell^{\mathsf{Sim}} \tag{16}$$

In order to show that the tampered codeword is independent of the message, we first need a statement similar to the inequality 16, but where the simulated output is replaced by the decoding of the tampering of $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c)$. We will obtain this using (15) twice and applying the triangle inequality. For the remainder of the proof, let $L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c$ be distributed as $L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c|_{(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_3}$. Also, let $\underline{K_w}, \underline{K_c}, \underline{S}, \underline{L}, \underline{R}, \underline{W}$ be the corresponding random

[8] For the purpose of this proof alone, we slightly abuse notation and let $\underline{S}, \underline{K_w}, \underline{K_c}$ denote the tampered seed and the MAC keys when the codeword $((L, W, \mathsf{Tag}_{k_w}(W)), (R, \widetilde{C}, \mathsf{Tag}_{k_c}(\widetilde{C})))$ is tampered using functions f, g.

variables after tampering conditioned on the event that $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_3$. We shorthand the distribution in the LHS and RHS of 16 by A and B, respectively. Also, we define $\mathsf{Vrfy}_\ell := \mathsf{Vrfy}_{\underline{K_w}}(f_2(L, W, \sigma_w), f_3(L, W, \sigma_w))$. Applying Lemma 2 on inequality 15, we get that

$$A \approx_{\frac{2\varepsilon}{\alpha}} \mathsf{Ext}(W; S), S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell \ , \tag{17}$$

and

$$B \approx_{\frac{2\varepsilon}{\alpha}} U, S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell \ . \tag{18}$$

By triangle inequality applied on the inequalities (17),(18), and (16), we have that

$$\mathsf{Ext}(W; S) \approx_{\varepsilon_2 + \frac{4\varepsilon}{\alpha}} U | S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell \ .$$

Notice that given $(S, L, \widetilde{C}, \underline{S}, \underline{K_w}, \underline{K_c}, \underline{L})$, the random variable R is independent of W, and hence we obtain that

$$\mathsf{Ext}(W; S) \approx_{\varepsilon_2 + \frac{4\varepsilon}{\alpha}} U | S, L, \widetilde{C}, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell, R, g_1(R, \widetilde{C}, \widetilde{\sigma}_c) \ .$$

Using Lemma 2, we can drop $\widetilde{C}$ on both sides to get

$$\mathsf{Ext}(W; S) \approx_{\varepsilon_2 + \frac{4\varepsilon}{\alpha}} U | S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell, R, \underline{R} \tag{19}$$

where $\underline{R} \stackrel{\text{def}}{=} g_1(R, \widetilde{C}, \widetilde{\sigma}_c)$.
We now observe that W is independent of $\widetilde{C}$ given $S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell, R, \underline{R}$. To see this, we will use Lemma 5 with $V_0 = L, R, S, k_w, k_c$, and then $V_1 = \underline{L}$, $V_2 = \underline{R}$, $V_3 = \mathsf{Ext}(\underline{W}; \underline{S}), \mathsf{Vrfy}_\ell$, and observing that $\underline{S}, \underline{K_w}, \underline{K_c}$ is a deterministic function of $\underline{L}$ and $\underline{R}$. Thus, $\mathsf{Ext}(W; S)$ is independent of $\widetilde{C}$ given $S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell, R, \underline{R}$.
Now, applying Lemma 6 to equation 19, with $X = \mathsf{Ext}(W, S)$, $Y = \widetilde{C}$, $Z = (S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell, R, \underline{R})$, and using the above observation that $Y = \widetilde{C}$ is independent of $\mathsf{Ext}(W; S)$ given Z and of U given Z, we get that conditioned on $\widetilde{C} \oplus \mathsf{Ext}(W; S) = m$ on the LHS, and on $\widetilde{C} \oplus U = m$ on the RHS:

$$C, S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell, R, \underline{R} \approx_{\varepsilon_2 + \frac{4\varepsilon}{\alpha}} \widetilde{C}, S, L, \underline{L}, \mathsf{Ext}(\underline{W}; \underline{S}), \underline{S}, \underline{K_w}, \underline{K_c}, \mathsf{Vrfy}_\ell, R, \underline{R} \ ,$$

since $\widetilde{C}$ conditioned on $\widetilde{C} \oplus U = m$ is distributed identically as $\widetilde{C}$ given all other random variables on the RHS in the above inequality.
The desired result follows by observing that the tampered codeword is a function of

$$\underline{L}, \underline{R}, \mathsf{Ext}(\underline{W}; \underline{S}), C, R, \mathsf{Vrfy}_\ell \ .$$

□

Proof of Theorem 3. The simulator $\mathsf{Sim}_{f,g}$ does the following. It samples $W, S, K_w, K_c, \sigma_w, L, R, \widetilde{C}, \widetilde{\sigma}_c$. Let $I^{\mathsf{sim}} \in \{1, 2, 3\}$ be a random variable indicating the partition $(\mathcal{P}_{I^{\mathsf{sim}}})$ in which the sampled codeword belongs. If $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_1$, then the simulator outputs $(\mathsf{same}^*, R, \widetilde{C}, \widetilde{\sigma}_c)$ and sets $I^{\mathsf{sim}} = 1$, if $(L, W, R, \widetilde{C}) \in \mathcal{P}_2$ then the simulator outputs $(\perp, R, \widetilde{C}, \widetilde{\sigma}_c)$ and sets $I^{\mathsf{sim}} = 2$, else the simulator outputs $\Big(\mathsf{Dec}((L, W, \sigma_w), (R, \widetilde{C}, \widetilde{\sigma}_c)), R, \widetilde{C}, \widetilde{\sigma}_c\Big)$ and sets $I^{\mathsf{sim}} = 3$.

Now, if $\Delta\left((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_i}\ ;\ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=i}\right) \leq \gamma_i$, by Lemma 4, we will get:

$$\Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c);\ D^m_{f,g}) \leq \sum_{i=1}^{3} \gamma_i \Pr[(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_i], \tag{20}$$

where $D^m_{f,g}$ is a distribution on $(\{0,1\}^{\kappa^*} \cup \{\mathsf{same}^*, \perp\}) \times \{0,1\}^{n+\kappa^*+t}$, such that $\Pr[D^m_{f,g} = d] = \sum_{i=1}^{3} \Pr[(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_i] \cdot \Pr[\mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=i} = d]$. But, by Lemma 10, we get $(C, \sigma_c) \approx_{4\varepsilon+2^{-\Omega(\kappa/\log\kappa)}} (\widetilde{C}, \widetilde{\sigma}_c)|(L, R, \underline{L}, b_1)$, where b_1 is the bit indicating if $(\underline{W}, \underline{\sigma_w}) = (W, \sigma_w)$ or not, and the event that $(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_i$ (or correspondingly $(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_i$) can be determined by the random variables on the LHS and RHS of this inequality. Hence, together with the fact that $\Pr[\mathsf{Copy}(m, \mathsf{Sim}_{f,g}) = d] = \sum_{i=1}^{3} \Pr[(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_i] \cdot \Pr[\mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=i} = d]$, for each $d \in (\{0,1\}^{\kappa^*} \cup \{\mathsf{same}^*, \perp\}) \times \{0,1\}^{n+\kappa^*+t}$, we get:

$$\Delta(D^m_{f,g}\ ;\ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})) \leq 4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)} \tag{21}$$

Combining inequality 20 and 21, by triangle inquality, we get:

$$\Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c);\ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})) \leq 4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)} + \sum_{i=1}^{3} \gamma_i \Pr[(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_i], \tag{22}$$

Hence, now we consider each partition and bound the corresponding term in the RHS of the summation in the inequality 22.
First, since we know that $(\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_1} = (m, R, C, \sigma_c)$ and $\mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=1} = (m, R, \widetilde{C}, \widetilde{\sigma}_c)$, we use Lemma 10 to get:

$$\begin{aligned}\Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_1};\ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=1}) &\leq \Delta((C, \sigma_c)\ ;\ (\widetilde{C}, \widetilde{\sigma}_c)|(L, R, \underline{L}, b_1)) \\ &\leq 4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)}\end{aligned} \tag{23}$$

Hence, $\gamma_1 = 4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)}$, on the RHS of inequality 22.
Similarly, since we know that by Lemma 11, $\Pr[(\mathsf{Tamper}^m_{f,g} \neq \perp \wedge (L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_2] \leq 4\varepsilon + 2^{-\Omega(\kappa)}$, and $\mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=2} = (\perp, R, \widetilde{C}, \widetilde{\sigma}_c)$, we use Lemma 10 to get:

$$\begin{aligned}&\Pr[(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_2] \cdot \Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_2};\ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=2}) \\ &\leq \Pr[(\mathsf{Tamper}^m_{f,g} \neq \perp \wedge (L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_2] + \Delta((C, \sigma_c)\ ;\ (\widetilde{C}, \widetilde{\sigma}_c)|(L, R, \underline{L}, b_1)) \\ &\leq 8\varepsilon + 2^{-\Omega(\kappa/\log\kappa)}\ .\end{aligned} \tag{24}$$

Hence, $\gamma_2 \cdot \Pr[(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_2] \leq 8\varepsilon + 2^{-\Omega(\kappa/\log\kappa)}$, on the RHS of inequality 22.
Now, for the third partition, we set $\alpha = \sqrt{\varepsilon} + 2^{-d/2} (> 2\varepsilon + 2^{-d})$ in Lemma 12, and get that for any fixing a, b of K_w, K_c, if $\Pr[(L, W, \sigma_w, R, \widetilde{C}, \widetilde{\sigma}_c) \in \mathcal{P}_3 | K_w = a, K_c = b] \geq \alpha = \sqrt{\varepsilon} + 2^{-d/2}$, then

$$\begin{aligned}&\Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_3, K_w=a, K_c=b}\ ;\ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=3, K_w=a, K_c=b}) \\ &\leq 2^{-\Omega(\kappa/\log\kappa)} + \frac{4\varepsilon}{\alpha} \\ &\leq 2^{-\Omega(\kappa/\log\kappa)} + 4\sqrt{\varepsilon}\end{aligned}$$

Now, since $d = \Omega(\kappa/\log\kappa)$, and once again using Lemma 10, we get:

$$\begin{aligned}
&\Pr[(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_3] \cdot \Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_3} \ ; \ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=3}) \\
&\leq 4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)} + \Pr[I^{\mathsf{sim}} = 3] \cdot \sum_{a,b} \Pr[K_w = a, K_c = b | I^{\mathsf{sim}} = 3] \\
&\qquad \cdot \Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{I^{\mathsf{sim}}=3,K_w=a,K_c=b} \ ; \ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=3,K_w=a,K_c=b}) \\
&\leq (4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)}) + \sum_{a,b} \Pr[K_w = a, K_c = b] \cdot \Pr[I^{\mathsf{sim}} = 3 | K_w = a, K_c = b] \\
&\qquad \cdot \Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{I^{\mathsf{sim}}=3,K_w=a,K_c=b} \ ; \ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})|_{I^{\mathsf{sim}}=3,K_w=a,K_c=b}) \\
&\leq (4\varepsilon + 2^{-\Omega(\kappa/\log\kappa)}) + \sum_{a,b} \Pr[K_w = a, K_c = b] \cdot O(\sqrt{\varepsilon} + 2^{-\Omega(\kappa/\log\kappa)}) \\
&\leq O(\sqrt{\varepsilon} + 2^{-\Omega(\kappa/\log\kappa)}) \qquad (25)
\end{aligned}$$

In the above inequality 25, we make an abuse of notation, and use $(\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{I^{\mathsf{sim}}=3,K_w=a,K_c=b}$ to denote the distribution where the partition is first picked using I^{sim}, and then the distribution $(\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)$ is drawn conditioned on the partition. This distribution is clearly identical to $(\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c)|_{(L,W,\sigma_w,R,C,\sigma_c)\in\mathcal{P}_3}$, which is what we use above.
Hence, $\gamma_3 \cdot \Pr[(L, W, \sigma_w, R, C, \sigma_c) \in \mathcal{P}_3] = O(\sqrt{\varepsilon} + 2^{-\Omega(\kappa/\log\kappa)})$, on the RHS of inequality 22. Finally, we can use inequalities 23,24 and 25, in inequality 22 to get:

$$\Delta((\mathsf{Tamper}^m_{f,g}, R, C, \sigma_c); \ \mathsf{Copy}(m, \mathsf{Sim}_{f,g})) \leq O(\sqrt{\varepsilon} + 2^{-\Omega(\kappa/\log\kappa)})$$

□

4 Application to Non-malleable Commitments

In this section, we show an application of our 1/3-rate NMC to get the first non-malleable commitment scheme with computational binding and statistical hiding, achieving a communication cost of approximately 41 times the length of the message being committed to. We begin by defining non-malleable commitments, introduced by Dolev, Dwork and Naor [DDN91], that give computational binding and statistical hiding property.

Definition 6. *[GPR16] A non-malleable commitment scheme, $\langle \mathcal{C}, \mathcal{R} \rangle$ is a two-phase, two-party protocol between a committer $\mathcal{C}$ and a receiver $\mathcal{R}$. In the commit phase, $\mathcal{C}$ uses secret m and interacts with $\mathcal{R}$ who uses no input. Let $z = \mathsf{Com}(m; r)$ denote $\mathcal{R}$'s view after the commit phase. Let $(w, m) = \mathsf{Decom}(z, m, r)$ denote $\mathcal{R}$'s view after the decommit phase, which $\mathcal{R}$ either accepts or rejects. We say that $\langle \mathcal{C}, \mathcal{R} \rangle$ is a computationally binding and ϵ-statistically hiding non-malleable commitment scheme if the following properties hold:*

1. ***Correctness:*** *If the parties follow the protocol, then $\mathcal{R}(z, w, m) = 1$, i.e., the receiver accepts.*
2. ***Binding:*** *For any PPT adversarial receiver $\mathcal{R}^*$, that outputs $(w', m'), (w, m), z$, with $m' \neq m$, the probability that $\mathcal{R}(z, w, m) = 1 = \mathcal{R}(z, w', m')$ is negligible.*
3. ***Hiding:*** *For all distinct message pairs m, m', $\{\mathsf{Com}(m; r)\}_r \approx_\epsilon \{\mathsf{Com}(m'; r')\}_{r'}$.*
4. ***Non-malleability:*** *For avoiding trivial man-in-the-middle attack of copying the identity of the committer, we consider the committer and receiver to additionally have an identity*

$\mathsf{id} \in \{0,1\}^\lambda$ *as common input (λ is the computational security parameter). To define non-malleability, we consider the real/ideal paradigm. In the real interaction, there is a man-in-the-middle adversary M interacting with a committer, $\mathcal{C}$, in the left session and a receiver $\mathcal{R}$, in the right. All the quantities associated with the right interaction are denoted by the "tilde'd" versions of their left counterparts (e.g., $\mathcal{C}$ commits to m in the left interaction while M commits to $\tilde{m}$ in the right). Let MIM_m denote the random variable describing $(\mathsf{VIEW}, \tilde{m})$, consisting of M's view in the experiment and the value M commits to in the right interaction, given that $\mathcal{C}$ committed to m on the left. The ideal interaction is the same, except that $\mathcal{C}$ commits to an arbitrary message, say 0, on the left. Let MIM_0 denote the corresponding random variable for 0. M is forced to use an identity $\tilde{\mathsf{id}}$ on the right, which is distinct from id used on the left. MIM_m and MIM_0 output a special symbol $\perp_{\mathsf{id}}$ when M has used the same identity on the right as received on the left.*
Non-malleability guarantees that for every PPT man-in-the-middle M, and for all messages m, we have $\{\mathsf{MIM}_m(y)\}_{y\in\{0,1\}^*} \approx_c \{\mathsf{MIM}_0(y)\}_{y\in\{0,1\}^*}$, where y is the auxiliary input received by M.

The round complexity of a commitment scheme denotes the number of rounds of interaction between the committer and receiver. The communication complexity of a commitment scheme denotes the total size of the transcript of the interaction between the committer and the receiver.

We consider the textbook non-malleable commitment scheme from [GPR16], which uses a 2-split-state augmented non-malleable code tolerating leakage, as an underlying building block. We instantiate their scheme with our 1/3-rate augmented non-malleable code to get a non-malleable commitment scheme with a communication complexity of $41 \cdot |\mathsf{message\ length}|$. To be able to use our NMC, we show that it tolerates leakage as well. We begin by looking at the building blocks used.

4.1 Building Blocks

The construction from [GPR16] requires two building blocks, which we instantiate using different schemes than theirs.

- A non-interactive computationally binding and statistically hiding commitment, $(\mathsf{Com}, \mathsf{Decom})$, with message space $\{0,1\}^{2\beta_1}$, which is a non-interactive two phase protocol as in Definition 6 satisfying correctness, computational binding and statistical hiding.

  We can instantiate our scheme with the hashing based statistically hiding commitment of [HM96], which has commitment size of $\approx 9 \cdot (\mathsf{message\ length})$.

- A leakage resilient and augmented non-malleable code, $(\mathsf{Enc}, \mathsf{Dec})$, with message space $\{0,1\}^\alpha$ and codeword space $\{0,1\}^{\beta_1} \times \{0,1\}^{\beta_2}$, as defined below:

  Definition 7 (Augmented Leakage Resilient Non-malleable codes)**.** *A coding scheme $(\mathsf{Enc}, \mathsf{Dec})$ from $\{0,1\}^\alpha$ to $\{0,1\}^{\beta_1} \times \{0,1\}^{\beta_2}$ is called an ϵ-augmented leakage resilient non-malleable code with respect to $\mathcal{F}_{\mathsf{split}}$, tolerating a μ bits of leakage from the left state, if the following holds. For any leakage function $\mathsf{Leak} : \{0,1\}^{\beta_1} \to \{0,1\}^\mu$ and any (possibly randomized) $(f,g) \in \mathcal{F}_{\mathsf{split}}$, let $\mathsf{Tamper}^m_{f,g}$ denote the distribution $\mathsf{Dec}(f(L), g(R))$, for $(L,R) \leftarrow \mathsf{Enc}(m)$, and $\mathsf{leak}_L = \mathsf{Leak}(L)$. There exists a simulator that, given access to $\mathsf{leak}_L = \mathsf{Leak}(L)$ and $\mathsf{Leak}(.)$, outputs a distribution $Sim_{f,g,\mathsf{leak}_L,\mathsf{Leak}}$ over $(\{0,1\}^\alpha \cup \{same^*, \perp\}) \times \{0,1\}^{\beta_2}$ such that, for all $m \in \{0,1\}^\alpha$*

$$\mathsf{Tamper}^m_{f,g}, R, \mathsf{leak}_L \approx_\epsilon \mathsf{Copy}(m, Sim_{f,g,\mathsf{leak}_L,\mathsf{Leak}}), \mathsf{leak}_L,$$

where $(\tilde{m}, R^{sim}) \leftarrow Sim_{f,g,\mathsf{leak}_L,\mathsf{Leak}}$*, and* $\mathsf{Copy}(m, Sim_{f,g,\mathsf{leak}_L,\mathsf{Leak}})$ *is defined as*

$$\mathsf{Copy}(m, Sim_{f,g,\mathsf{leak}_L,\mathsf{Leak}}) \stackrel{def}{\equiv} \begin{cases} (m, R^{sim}) \text{ if } \tilde{m} = \mathsf{same}^* \\ (\tilde{m}, R^{sim}) \text{ otherwise} \end{cases}$$

We can directly use [BFO+20, Theorem 3] to show that our augmented (w.r.t. the right state) NMC from Theorem 3 is also tolerant to μ bits of leakage from the left state, as in Definition 7. This is formally stated in the lemma below, and we give a complete proof of the same in Appendix A.

Lemma 13. *If* $(\mathsf{Enc}, \mathsf{Dec})$ *is a 2-split-state* ϵ*-augmented non-malleable code, then it is also a 2-split-state* $2^{\mu}\epsilon$*-augmented leakage resilient non-malleable code, tolerating* μ *bits of leakage from the left state, as in Definition 7.*

4.2 Construction

We now describe the construction of non-malleable commitments from [GPR16], using the building blocks from Section 4.1. For multiplication and addition operations in the construction below, we assume a natural correspondence between the binary β_1-bit strings and the field $GF(2^{\beta_1})$.

- **Setup:** Let $\mathsf{id} \in \{0,1\}^{\lambda}$ be $\mathcal{C}$'s identity, also given as input to $\mathcal{R}$. λ is the computational security parameter.
- **Inputs:** $\mathcal{C}$ has input message $m \in \{0,1\}^{\alpha}$ to be committed to. id is a common input of both $\mathcal{C}$ and $\mathcal{R}$.
- **Commit Phase:**
  1. $\mathcal{C} \rightarrow \mathcal{R}$: Let $(L, R) \leftarrow \mathsf{Enc}(m \| \mathsf{id})$. Pick random $r \leftarrow \{0,1\}^{\beta_1}$ and send $\mathsf{Com}(L \| r)$ to $\mathcal{R}$.
  2. $\mathcal{R} \rightarrow \mathcal{C}$: Send random $a \leftarrow \{0,1\}^{\beta_1} \backslash \{0^{|\beta_1|}\}$.
  3. $\mathcal{C} \rightarrow \mathcal{R}$: Send $b = ra + L$ and R.
- **Decommit Phase:** $\mathcal{C}$ opens the commitment in Step 1. Let $L' \| r'$ be the decommited value.
- **Receiver's Output:** If L' and r' do not satisfy $r'a + L' = b$, then output $\perp_{inc}$. Else, compute $m' \| \mathsf{id}' = \mathsf{Dec}(L', R)$, and output $\perp_{\mathsf{id}}$ if $\mathsf{id}' = \mathsf{id}$. Else output m'.

Figure 5: Non-malleable Commitment Scheme $\langle \mathcal{C}, \mathcal{R} \rangle$

In [GPR16], the additional property needed from the underlying NMC is called conditional augmented property [GPR16, Definition 10], which guarantees that if the left state L is first picked at random from the space of left state of valid codewords (whose decode is $\neq \perp$) and then the right state is picked, conditioned on the message and the left state, the augmented non-malleability (with right augmentedness) is still guaranteed. We observe here that the proof of [GPR16, Claim 2], showing that a non-malleable code is conditional augmented, only requires leakage resilience from the left state of the NMC. Hence, we can re-state the main theorem of [GPR16, Theorem 1], with instantiations from Section 4.1, as follows.

Theorem 4. *[GPR16] If* $(\mathsf{Com}, \mathsf{Decom})$ *is a non-interactive computationally binding and statistically hiding commitment scheme, and* $(\mathsf{Enc}, \mathsf{Dec})$ *is a leakage resilient augmented non-malleable code, then the protocol* $\langle \mathcal{C}, \mathcal{R} \rangle$ *in Figure 5 is a non-malleable commitment scheme against synchronizing adversary with computational binding and statistical hiding.*

Further, using the hashing based non-interactive commitment scheme [HM96] and our rate $1/3$*-augmented and leakage resilient non-malleable code, the communication cost of the above scheme is* 41α*, where* α *is the message length.*

Proof. We refer the reader to [GPR16] for a complete proof of the theorem above. The only difference is that, since we use a computational binding and statistical hiding commitment scheme to instantiate, we also get computational binding (which directly follows from the binding property of the underlying commitment scheme), and statistical hiding (which directly follows from the hiding property of the underlying commitment scheme and secret sharing property of the non-malleable code) for the non-malleable commitment scheme.

Communication Cost. Since the commitment size of $(\mathsf{Com}, \mathsf{Decom})$ is 9 times the length of the message being committed, $|L| = 2\alpha$ and $|R| = \alpha$, we get the total size of the transcript $= |\mathsf{Com}(L\|r)| + |a| + |b| + |R| \approx 9(|L| + |r|) + 5\alpha = 41\alpha$. □

5 A Rate Booster for Two-source Non-malleable Extractors

We begin by defining a strong two-source non-malleable extractor, introduced in [CG14b].

Definition 8. *A function* $\mathsf{nm2Ext} : \{0,1\}^{n_1} \times \{0,1\}^{n_2} \to \{0,1\}^m$ *is a strong* $(n_1, n_2, k_1, k_2, \epsilon)$*-strong two-source non-malleable extractor if for each* (n_1, k_1)*-source* X *and* (n_2, k_2)*-source* Y*, the following holds. Let* $f : \{0,1\}^{n_1} \to \{0,1\}^{n_1}$ *and* $g : \{0,1\}^{n_2} \to \{0,1\}^{n_2}$ *be functions with no fixed points, i.e., for each* $x \in \{0,1\}^{n_1}$*,* $y \in \{0,1\}^{n_2}$*,* $f(x) \neq x$ *and* $g(y) \neq y$*. Then,*

$$\mathsf{nm2Ext}(X, Y), X, \mathsf{nm2Ext}(f(X), g(Y)) \approx_\epsilon U_m, X, \mathsf{nm2Ext}(f(X), g(Y))$$

We drop the adjective strong when the distribution X *is excluded from the statistical distance above. The rate is defined to be the ratio* $m/(n_1 + n_2)$*.*

We consider an $(n_1, n_2, k_1, k_2, d, \epsilon_1)$-strong two-source unbalanced non-malleable extractor, $\mathsf{nm2Ext}$, with $n_2 = o(n_1)$, and an $(n_1, k_1, d, l, \epsilon_2)$-strong seeded extractor and define the following function $\mathsf{nm2Ext}^* : \{0,1\}^{n_1} \times \{0,1\}^{n_2} \to \{0,1\}^l$:

$$\mathsf{nm2Ext}^*(x, y) \stackrel{\text{def}}{=} \mathsf{Ext}(x; \mathsf{nm2Ext}(x, y)),\ \forall\ x \in \{0,1\}^{n_1}, y \in \{0,1\}^{n_2}$$

We show that the above function is a two-source non-malleable extractor achieving rate $1/2$, with appropriate instantiation of Ext.

Theorem 5. *If* $\mathsf{nm2Ext}$ *is an* $(n_1, n_2, k_1, k_2, d, \epsilon_1)$*-strong two-source unbalanced non-malleable extractor, with* $n_2 = o(n_1)$ *and* Ext *is an* $(n_1, k_1, d, l, \epsilon_2)$*-strong seeded extractor,then* $\mathsf{nm2Ext}^*$ *is an* $(n_1, n_2, k_1, k_2, l, \epsilon_1 + \epsilon_2)$*-two-source non-malleable extractor. Further, if we instantiate* Ext *with the seeded extractor in Lemma 7, with* $k_1, l < n_1/2$*, then the rate of* $\mathsf{nm2Ext}^*$ *is* $1/2$*.*

Proof. Consider an (n_1, k_1)-source X, an (n_2, k_2)-source Y and tampering functions f and g (with no fixed points). By the $\mathsf{nm2Ext}$ security, we get:

$$X, \mathsf{nm2Ext}(X, Y), \mathsf{nm2Ext}(f(X), g(Y)) \approx_{\epsilon_1} X, U_d, \mathsf{nm2Ext}(f(X), g(Y))$$

By applying the function Ext on the first two distributions on either side and using Lemma 2, we get:

$$\mathsf{Ext}(X; \mathsf{nm2Ext}(X,Y)), \mathsf{nm2Ext}(f(X), g(Y)) \approx_{\epsilon_1} \mathsf{Ext}(X; U_d), \mathsf{nm2Ext}(f(X), g(Y))$$

By security of Ext, since $\widetilde{\mathbf{H}}_\infty(X|\mathsf{nm2Ext}(f(X), g(Y))) \geq k_1$, we get:

$$\mathsf{Ext}(X; U_d), \mathsf{nm2Ext}(f(X), g(Y)) \approx_{\epsilon_2} U_l, \mathsf{nm2Ext}(f(X), g(Y))$$

Hence, by triangle inequality, this gives:

$$\mathsf{Ext}(X; \mathsf{nm2Ext}(X,Y)), \mathsf{nm2Ext}(f(X), g(Y)) \approx_{\epsilon_1+\epsilon_2} U_l, \mathsf{nm2Ext}(f(X), g(Y))$$

which proves that $\mathsf{nm2Ext}^*$ is a two-source non-malleable extractor.
Rate. Setting Ext to be the extractor from Lemma 7, with $k_1, l < n/2$ gives $\mathsf{nm2Ext}^*$ a rate of at most $(n_1/2)/(n_1+n_2) = 1/2$. □

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A Leakage Resilience of Our NMC: Proof of Lemma 13

We prove the following lemma (which follows from [BFO+20, Theorem 3]), which states that any augmented non-malleable code against $\mathcal{F}_{\mathsf{split}}$ also allows a single bit of leakage from one of the states, as needed for our instantiation.

Lemma 13. *If* $(\mathsf{Enc}, \mathsf{Dec})$ *is a* 2*-split-state* ϵ*-augmented non-malleable code, then it is also a* 2*-split-state* $2^{\mu}\epsilon$*-augmented leakage resilient non-malleable code tolerating* μ *bits of leakage from the left state, as in Definition 7.*

Proof. Suppose to the contrary that there exists an unbounded adversary $\mathcal{A}$ that breaks the augmented (w.r.t the right state R) non-malleability of $(\mathsf{Enc}, \mathsf{Dec})$, given the μ bits of leakage on the left state L. Hence, there exists a message m and tampering functions $(f, g) \in \mathcal{F}_{\mathsf{split}}$ and leakage function Leak such that

$$|\Pr[\mathcal{A}(\mathsf{Tamper}^m_{f,g}, R, \mathsf{leak}_L) = 1] = \Pr[\mathcal{A}(\mathsf{Copy}(m, Sim_{f,g,\mathsf{leak}_L,\mathsf{Leak}}), \mathsf{leak}_L) = 1]| > 2^{\mu}\epsilon,$$

where $\mathsf{leak}_L = \mathsf{Leak}(L)$, and $\mathsf{Tamper}^m_{f,g}$ and $\mathsf{Copy}(m, Sim_{f,g,\mathsf{leak}_L,\mathsf{Leak}})$ are as in Definition 7. Then, consider the following unbounded reduction $\mathcal{A}'$ that breaks the augmented non-malleability of $(\mathsf{Enc}, \mathsf{Dec})$, without the leakage:

- On input leakage query to Leak, the reduction $\mathcal{A}'$ picks a random $\mathsf{leak}^* \leftarrow \{0,1\}^{\mu}$ and sends it as the response.
- Further, on receiving the tampering functions (f, g), $\mathcal{A}'$ forwards it to the augmented NMC challenger, hard-wiring Leak and leak^*, where the tampering function acting on L first checks if $\mathsf{Leak}(L) = \mathsf{leak}^*$ and tampers only if it does, else outputs $\perp$. On receiving the response $(R^b, \tilde{m}^b)$ from the challenger, $\mathcal{A}'$ forwards it to the adversary $\mathcal{A}$, along with leak^*.
- Output the same guessing bit as $\mathcal{A}$.

Clearly, when the random string leak^* matches the actual leakage bit on L, which happens with probability $1/2^{\mu}$, the reduction sends the correctly simulated distributions to $\mathcal{A}$. Hence, the winning probability of $\mathcal{A}$ is exactly translated to the winning probability of $\mathcal{A}'$ with probability $1/2^{\mu}$. This gives us

$$|\Pr[\mathcal{A}'(\mathsf{Tamper}^m_{f,g}, R) = 1] - \Pr[\mathcal{A}'(\mathsf{Copy}(m, Sim_{f,g_{(\mathsf{Leak},\mathsf{leak}^*)}})) = 1]| > \epsilon$$

Hence, the proof of the lemma holds. □

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