

On the Fixed-Order Strong Komlós Conjecture

Shiva Kintali*

September 15, 2026

Abstract

The strong Komlós conjecture asserts that every ordered family of Euclidean-unit vectors admits a signing whose signed prefixes have uniformly bounded ℓ_∞ -norm. We disprove this conjecture by constructing explicit finite families with unbounded fixed-order prefix discrepancy. At level k , our integer matrix has $d_k = 2^{2^k - 1}$ rows and exactly $s_k = 2^k$ nonzero ± 1 entries per column. Every real coefficient assignment with magnitudes at least one produces a coordinate trajectory of range at least s_k ; after normalization, this yields prefix discrepancy at least $\frac{1}{2}\sqrt{s_k} = \frac{1}{2}\sqrt{1 + \log_2 d_k} \rightarrow \infty$. Our construction uses a nested-word amplification with detector columns. It also gives linear lower bounds for sparse binary matrices, square $N \times N$ examples with discrepancy $\Omega(\sqrt{\log \log N})$, and separations from ordinary discrepancy and freely reordered prefix discrepancy.

1 Introduction

For an ordered real matrix $A = (v_1, \dots, v_n)$ with columns $v_i \in \mathbb{R}^d$, define its *fixed-order prefix discrepancy*

$$\text{pdisc}(A) = \min_{\varepsilon \in \{-1, 1\}^n} \max_{0 \leq j \leq n} \left\| \sum_{i=1}^j \varepsilon_i v_i \right\|_\infty. \quad (1.1)$$

The signs in (1.1) may depend on the entire ordered matrix; this is an offline problem, and the input order cannot be changed. The *strong Komlós conjecture* asks whether there is an absolute constant C such that $\text{pdisc}(A) \leq C$ whenever $\|v_i\|_2 \leq 1$, independently of d and n . It was posed in this form, and under this name, by Bansal, Jiang, Meka, Singla, and Sinha [2, Open Problem 6.1 in the full version]; Karingula and Lovett use the same name [4]. Prefix balancing questions of this kind go back to Spencer [6].

Ordinary Komlós discrepancy controls only the final signed sum. Recent constant bounds for that problem [3, 4] allow large intermediate excursions and therefore do not imply a prefix bound; both works note that their methods do not control prefixes.

For fixed order, Banaszczyk’s theorem gives $\text{pdisc}(A) = O(\sqrt{\log d + \log n})$ [1], as recorded in [2, Theorem 1.1]; the standard row reduction gives the dimension-free form $O(\sqrt{\log n})$ [4, footnote 1]. We are not aware of any previously known lower bound for this offline problem that grows with d or n ; [2] records Banaszczyk’s bound as the best known result for Open Problem 6.1 and notes that no separation from ordinary Komlós discrepancy was known. Online discrepancy has a different information pattern: the optimal $O(\sqrt{\log n})$ online result [5] does not produce a single fixed ordered matrix that remains hard when the signing may depend on the entire input.

*Email: shiva.kintali@gmail.com Homepage: shivakintali.github.io

Our Results. Theorem 3.1 gives, for every integer $k \geq 0$, an explicit ordered matrix A_k with Euclidean-unit columns and $d_k = 2^{k-1}$ rows such that

$$\text{pdisc}(A_k) \geq 2^{k/2-1} = \frac{1}{2} \sqrt{1 + \log_2 d_k}.$$

Hence the strong Komlós conjecture is false. The conclusion remains valid if the signs are replaced by arbitrary real coefficients of magnitude at least one. The length grows extremely rapidly, so the construction is qualitative rather than efficient. Its most direct additional consequence is a linear lower bound for s -sparse binary matrices (Corollary 3.2), which rules out the anticipated $O(\sqrt{s})$ prefix analogue of Beck–Fiala. We also record a square $N \times N$ lower bound of $\Omega(\sqrt{\log \log N})$ and show that, on the same family, ordinary discrepancy and freely reordered prefix discrepancy tend to zero. Thus the obstruction is specifically one of fixed-order prefixes.

Our Method. Every output coordinate comes in a pair; we call the half-sum of a pair its mean and the half-difference its tracker. Original columns move only means, detector columns move only trackers, and a nested word freezes one large mean at each depth before running a detector for every possible tuple of rows. The amplification lemma (Lemma 2.2) doubles the guaranteed excursion at each level. In effect the construction converts an adaptive adversary into a fixed sequence by enumerating all of its possible responses, which is the source of the rapid growth.

2 The robust amplification construction

2.1 The offset-robust prefix property

Write $[d] = \{1, \dots, d\}$. For $c = (c_1, \dots, c_n) \in \mathbb{R}^n$, write

$$S_j(A, c) = \sum_{i=1}^j c_i v_i, \quad S_0(A, c) = 0,$$

and write $[x]_r$ for the r -th coordinate of a vector $x \in \mathbb{R}^d$. For every fixed coefficient assignment, translating the finite path coordinatewise gives

$$\inf_{w \in \mathbb{R}^d} \max_{0 \leq j \leq n} \|w + S_j(A, c)\|_\infty = \frac{1}{2} \max_{1 \leq r \leq d} \left(\max_{0 \leq j \leq n} [S_j(A, c)]_r - \min_{0 \leq j \leq n} [S_j(A, c)]_r \right). \quad (2.1)$$

Indeed, the maximum over j of $\|w + S_j\|_\infty$ is the largest over r of the maximum over j of $|w_r + [S_j]_r|$, the infimum over w decouples across coordinates, and each coordinate is optimally translated to the midpoint of its minimum and maximum. In particular the infimum in (2.1) is attained.

Definition 2.1. For a nonempty coefficient set $E \subseteq \mathbb{R}$ and $a \geq 0$, let $P_E(A, a)$ denote the statement

$$\text{for every } c \in E^n \text{ and } w \in \mathbb{R}^d, \quad \max_{0 \leq j \leq n} \|w + S_j(A, c)\|_\infty \geq a. \quad (2.2)$$

By (2.1), $P_E(A, a)$ is equivalent to every permitted coefficient assignment having some coordinate range at least $2a$. The arbitrary offset w is the key feature that allows sequential copies of the construction to reuse coordinates.

2.2 The nested word

Let $M = (m_{rj})$ be a $d \times n$ ordered matrix, with $d, n \geq 1$. The output coordinates are indexed by triples

$$(\ell, r, \sigma), \quad \ell, r \in [d], \quad \sigma \in \{+, -\}.$$

There are $2d^2$ coordinates. Let $e_{\ell, r, \sigma}$ be the corresponding coordinate vector.

For every depth $\ell \in [d]$ and source column $j \in [n]$, define an *original* column

$$O_{\ell, j} = \sum_{r=1}^d m_{rj} (e_{\ell, r, +} + e_{\ell, r, -}). \quad (2.3)$$

For every tuple $\rho = (r_1, \dots, r_d) \in [d]^d$, define a detector word $D_{\rho, 1}, \dots, D_{\rho, n}$ by

$$D_{\rho, j} = \sum_{\ell=1}^d m_{\ell j} (e_{\ell, r_\ell, +} - e_{\ell, r_\ell, -}). \quad (2.4)$$

In (2.4), the source row is ℓ ; the entry r_ℓ chooses only the physical coordinate pair into which that source row is placed. Different depths always use disjoint pairs. Thus an original column $O_{\ell, j}$ taken with coefficient c adds cm_{rj} to both coordinates of the pair (ℓ, r) , for every $r \in [d]$, and touches no other coordinate; a detector column $D_{\rho, j}$ taken with coefficient c adds $cm_{\ell j}$ to coordinate $(\ell, r_\ell, +)$ and subtracts $cm_{\ell j}$ from coordinate $(\ell, r_\ell, -)$, for every $\ell \in [d]$, and touches no other coordinate.

Let Q be the concatenation, in lexicographic order, of the d^d detector words. Define words $W_{d+1}, W_d, \dots, W_1$ backwards by

$$\begin{aligned} W_{d+1} &= Q, \\ W_\ell &= W_{\ell+1}, O_{\ell, 1}, W_{\ell+1}, O_{\ell, 2}, \dots, O_{\ell, n}, W_{\ell+1} \quad (\ell = d, \dots, 1). \end{aligned} \quad (2.5)$$

The output $F(M)$ is the matrix whose ordered column word is W_1 . Every physical occurrence in (2.5) receives its own coefficient: repeated column vectors do not share signs or coefficients. The word is completely fixed before those coefficients are chosen.

Every original and detector column corresponding to M_j satisfies

$$\|O_{\ell, j}\|_2^2 = \|D_{\rho, j}\|_2^2 = 2\|M_j\|_2^2. \quad (2.6)$$

There are no coordinate collisions in either sum. In particular, if every source column has exactly s nonzero entries, all equal to ± 1 , then every output column has exactly $2s$ such entries.

If n' and d' are the length and dimension of $F(M)$, then

$$d' = 2d^2, \quad n' = (1 + nd^d)(n + 1)^d - 1. \quad (2.7)$$

Indeed, $|W_{d+1}| = nd^d$ and $|W_\ell| = (n + 1)|W_{\ell+1}| + n$. Equivalently, $|W_\ell| + 1 = (1 + nd^d)(n + 1)^{d+1-\ell}$, which gives (2.7) at $\ell = 1$.

2.3 The amplification lemma

Lemma 2.2 (Nested mean-difference amplification). *If $P_E(M, a)$ holds, then $P_E(F(M), 2a)$ holds.*

Proof. Let n' be the length of $F(M)$. Fix arbitrary coefficients $c \in E^{n'}$ on the physical occurrences in $F(M)$, and fix an arbitrary initial offset $w \in \mathbb{R}^{2d^2}$. For $0 \leq t \leq n'$ let $p(t) = w + S_t(F(M), c)$, and for each pair of physical coordinates write

$$p_{\ell,r,+}(t) = h_{\ell,r}(t) + z_{\ell,r}(t), \quad p_{\ell,r,-}(t) = h_{\ell,r}(t) - z_{\ell,r}(t). \quad (2.8)$$

We call $h_{\ell,r}$ the *mean* and $z_{\ell,r}$ the *tracker* of the pair (ℓ, r) ; both are determined by c and w . By the description following (2.4), an original column at depth ℓ changes the depth- ℓ mean vector $h_{\ell,\cdot} = (h_{\ell,r})_{r \in [d]}$ by the coefficient of that physical occurrence times the corresponding source column, and changes no tracker and no mean at another depth. A detector column $D_{\rho,j}$ with coefficient β changes z_{ℓ,r_ℓ} by $\beta m_{\ell j}$ for every ℓ , and changes no mean and no other tracker.

Consider one occurrence of W_ℓ in the word, for some $\ell \in [d]$. By (2.5) it consists of $n+1$ child occurrences of $W_{\ell+1}$ separated by $O_{\ell,1}, \dots, O_{\ell,n}$, in that order, and these originals are the only columns in the occurrence that change $h_{\ell,\cdot}$. Therefore, if h_0 denotes the value of $h_{\ell,\cdot}$ at the start of the occurrence, the values of $h_{\ell,\cdot}$ at the starts of the $n+1$ child occurrences are

$$h_0, \quad h_0 + \alpha_1 M_1, \quad \dots, \quad h_0 + \sum_{j=1}^n \alpha_j M_j, \quad (2.9)$$

where $\alpha_j \in E$ is the coefficient of the occurrence of $O_{\ell,j}$ in question. This is an M -path with coefficients in E and offset h_0 , so $P_E(M, a)$ provides a row $r_\ell \in [d]$ and a slot $s \in \{0, \dots, n\}$ with

$$\left| [h_0 + \sum_{j \leq s} \alpha_j M_j]_{r_\ell} \right| \geq a. \quad (2.10)$$

Throughout the child occurrence of $W_{\ell+1}$ that starts at slot s , the mean h_{ℓ,r_ℓ} keeps this value, because that child contains only detectors and originals of depth greater than ℓ .

Begin with the single occurrence of W_1 , select a child occurrence of W_2 as above, and repeat inside the selected child. After d steps this produces one occurrence of $W_{d+1} = Q$ and rows $r_1, \dots, r_d$ such that throughout that occurrence of Q ,

$$|h_{\ell,r_\ell}(t)| \geq a \quad (\ell \in [d]), \quad (2.11)$$

each of these d means being constant there: the selected Q lies inside the child selected at every depth, and Q itself contains no originals. The selection is a post hoc choice of a witness interval in the already fixed word; it does not alter the word or its coefficients.

The selected occurrence of Q contains the detector word indexed by $\rho = (r_1, \dots, r_d)$. During that word, the vector $(z_{1,r_1}, \dots, z_{d,r_d})$ changes by $\beta_j M_j$ at the column $D_{\rho,j}$ with coefficient $\beta_j \in E$, so it traces an M -path with coefficients in E and some offset. Earlier detectors may have changed this offset, which is harmless because P_E permits every offset. The initial point of the detector is included as its prefix zero. Hence, at some prefix of this detector word, some $\ell \in [d]$ satisfies

$$|z_{\ell,r_\ell}(t)| \geq a. \quad (2.12)$$

At the same time t , (2.11) holds. The scalar identity $\max\{|h+z|, |h-z|\} = |h| + |z|$ then shows that one of the two physical coordinates of the pair (ℓ, r_ℓ) has magnitude at least $2a$ at time t , that is, $\|p(t)\|_\infty \geq 2a$. The coefficient assignment and initial offset were arbitrary, which is exactly $P_E(F(M), 2a)$. Notice that no detector must start or end at zero: offset robustness absorbs the effect of earlier copies, while ancestor means remain frozen on the selected descendant interval and different depths use disjoint coordinate pairs. $\square$

3 Counterexamples

3.1 The main counterfamily

Theorem 3.1. *For every integer $k \geq 0$, there is an explicitly specified finite ordered integer matrix M_k with $d_k = 2^{2^k-1}$ rows, $s_k = 2^k$ nonzero entries in every column, all belonging to $\{-1, 1\}$, such that every real coefficient assignment $|c_i| \geq 1$ has a coordinate whose partial-sum trajectory, including prefix zero, has range at least s_k . Consequently, $A_k = M_k/\sqrt{s_k}$ has Euclidean-unit columns and*

$$\text{pdisc}(A_k) \geq \frac{\sqrt{s_k}}{2} = 2^{k/2-1} = \frac{1}{2}\sqrt{1 + \log_2 d_k} \rightarrow \infty. \quad (3.1)$$

Proof. Let $E = \{c \in \mathbb{R} : |c| \geq 1\}$, take $M_0 = [1]$, and define $M_{k+1} = F(M_k)$. For $c \in E$, $\max\{|w|, |w + c|\} \geq |c|/2 \geq 1/2$, so $P_E(M_0, 1/2)$ holds. Lemma 2.2 gives

$$P_E(M_k, 2^{k-1}). \quad (3.2)$$

By (2.1), every permitted path therefore has maximum coordinate range at least 2^k .

Equations (2.6) and (2.7) give

$$s_{k+1} = 2s_k, \quad d_{k+1} = 2d_k^2, \quad s_0 = d_0 = 1.$$

Thus $s_k = 2^k$. Writing $e_k = \log_2 d_k$ gives $e_{k+1} = 1 + 2e_k$ and $e_0 = 0$, hence $d_k = 2^{2^k-1}$. Every recursively specified length is finite. Dividing by $\sqrt{s_k}$ makes every column have Euclidean norm exactly one. Applying (3.2) with zero offset and coefficients $\varepsilon_i \in \{-1, 1\}$ proves (3.1). Since the lower bound is positive, its witness cannot be the empty prefix. $\square$

The construction is explicit and finite, although its length grows extremely rapidly.

3.2 Sparse instances

The unnormalized matrices also answer the prefix version of the Beck–Fiala problem in the negative. In [2] it is asked whether s -sparse inputs admit a prefix bound analogous to the Beck–Fiala bound $O(s)$, and it is noted that a positive answer to their Open Problem 6.1 would give $O(\sqrt{s})$.

Corollary 3.2 (Sparse matrices). *For every integer $k \geq 0$, the unnormalized matrix M_k has entries in $\{-1, 0, 1\}$, exactly $s_k = 2^k$ nonzero entries in every column, and*

$$\text{pdisc}(M_k) \geq \frac{s_k}{2}. \quad (3.3)$$

Splitting every row into its positive and negative parts gives a matrix $\widetilde{M}_k$ with entries in $\{0, 1\}$, $2d_k$ rows, exactly s_k nonzero entries in every column, and $\text{pdisc}(\widetilde{M}_k) \geq s_k/4$. Consequently, for every integer $s \geq 1$ there is a matrix with entries in $\{0, 1\}$ and at most s nonzero entries per column whose fixed-order prefix discrepancy exceeds $s/8$.

Proof. Theorem 3.1 gives, for every signing, a coordinate of the unnormalized path whose range is at least s_k . Since the path starts at zero, that coordinate has magnitude at least $s_k/2$ at some prefix, which is (3.3). Write $M_k = M_k^+ - M_k^-$ with $M_k^\pm$ having entries in $\{0, 1\}$ and disjoint supports, and let $\widetilde{M}_k$ be M_k^+ stacked over M_k^- . Every column of $\widetilde{M}_k$ has exactly s_k ones. For every signing ε and prefix j ,

$$\left\| \sum_{i \leq j} \varepsilon_i (M_k)_i \right\|_\infty \leq \left\| \sum_{i \leq j} \varepsilon_i (M_k^+)_i \right\|_\infty + \left\| \sum_{i \leq j} \varepsilon_i (M_k^-)_i \right\|_\infty \leq 2 \left\| \sum_{i \leq j} \varepsilon_i (\widetilde{M}_k)_i \right\|_\infty,$$

so $\text{pdisc}(\widetilde{M}_k) \geq \text{pdisc}(M_k)/2 \geq s_k/4$. For the last claim, given an integer $s \geq 1$, choose k with $2^k \leq s < 2^{k+1}$; then $\widetilde{M}_k$ has at most s nonzero entries per column and $\text{pdisc}(\widetilde{M}_k) \geq 2^k/4 > s/8$. $\square$

Thus the fixed-order prefix discrepancy of s -sparse binary matrices is not $O(\sqrt{s})$, and not $o(s)$; the known upper bounds $\min\{O(d), O(\sqrt{s \log(dn)})\}$ recorded in [2] are of order s or larger on these instances, because $\log_2 d_k = s_k - 1$.

3.3 Square instances

For completeness, the construction can be padded to square matrices.

Corollary 3.3 (Square matrices). *For every integer $N \geq 1$, there is an $N \times N$ real matrix B_N with Euclidean-unit columns such that*

$$\text{pdisc}(B_N) \geq \frac{1}{2\sqrt{2}} \sqrt{1 + \log_2 \log_2(N+1)}. \quad (3.4)$$

Proof. Let n_k be the number of columns of M_k . We claim

$$n_k + 1 \leq 2^{d_k}. \quad (3.5)$$

We prove (3.5) simultaneously with $d_k \leq n_k$. Both hold with equality for $k = 0$. For the induction step, set

$$n = n_k, \quad d = d_k, \quad n' = n_{k+1}, \quad d' = d_{k+1}.$$

If $n + 1 \leq 2^d$, then (2.7) gives

$$\begin{aligned} n' + 1 &= (1 + nd^d)(n + 1)^d \\ &\leq d^d (n + 1)^{d+1} \\ &\leq 2^{d \log_2 d + d(d+1)} \leq 2^{2d^2} = 2^{d'}, \end{aligned}$$

where the first inequality uses $1 + nd^d \leq (n + 1)d^d$, and the last is equivalent to $\log_2 d + 1 \leq d$. Also, if $n \geq d$, then the recurrence gives $n' \geq nd(n + 1) \geq 2d^2 = d'$, so both assertions propagate. The lengths increase without bound.

Choose k with $n_k \leq N < n_{k+1}$. By (3.5), $N + 1 \leq n_{k+1} < n_{k+1} + 1 \leq 2^{d_{k+1}}$, and therefore

$$1 + \log_2 \log_2(N + 1) \leq 1 + \log_2 d_{k+1} = 2s_k. \quad (3.6)$$

Pad A_k with zero rows to dimension N , which is possible because $d_k \leq n_k \leq N$, then append $N - n_k$ copies of the unit coordinate column e_1 . Every signing of the resulting $N \times N$ matrix B_N restricts to a signing of its initial A_k block, and the prefixes of that block are among the prefixes of B_N , so $\text{pdisc}(B_N) \geq \text{pdisc}(A_k)$. Combining (3.1) and (3.6) proves the result. $\square$

4 Model separation

For an ordered matrix $A = (A_1, \dots, A_n)$, let

$$\text{disc}(A) = \min_{\varepsilon \in \{-1, 1\}^n} \left\| \sum_{i=1}^n \varepsilon_i A_i \right\|_{\infty}$$

denote its ordinary final-sum discrepancy.

Proposition 4.1. *Let n_k be the number of columns of the basic family $A_k = M_k/\sqrt{s_k}$. Then*

$$\text{disc}(A_k) = \frac{1}{\sqrt{s_k}}, \quad (4.1)$$

and its freely reordered prefix discrepancy is

$$\min_{\substack{\pi \in \mathfrak{S}_{n_k} \\ \varepsilon \in \{-1,1\}^{n_k}}} \max_{0 \leq j \leq n_k} \left\| \sum_{i=1}^j \varepsilon_i(A_k)_{\pi(i)} \right\|_{\infty} = \frac{1}{\sqrt{s_k}}. \quad (4.2)$$

Proof. We first prove simultaneously by induction on k that:

- (a) n_k is odd;
- (b) for $j_k = (n_k + 1)/2$, there is a nonnegative row vector f_k , supported on exactly s_k rows and equal to $1/s_k$ on its support, such that

$$f_k^T M_k = e_{j_k}^T; \quad (4.3)$$

where e_{j_k} is the j_k -th standard basis vector of $\mathbb{R}^{n_k}$; in particular, the entries of f_k sum to one;

- (c) among the column types of M_k , where two columns have the same type when they are equal as vectors, exactly one type has odd multiplicity, namely the type of the central column $(M_k)_{j_k}$.

For $M_0 = [1]$, all three assertions hold with $f_0 = (1)$.

Suppose they hold for the source matrix $M = M_k$, and write $d = d_k$, $n = n_k$, $s = s_k$, and $j = (n + 1)/2$. Let f be the vector in (b). By (2.7), $n' + 1 = (1 + nd^d)(n + 1)^d$ is even, so the output length n' is odd. Define g on the output coordinates by

$$g_{1,r,+} = g_{1,r,-} = f_r/2, \quad g_{\ell,r,\sigma} = 0 \quad (\ell > 1).$$

Then g is nonnegative, has total mass one, and is supported on $2s$ rows with value $1/(2s)$ on its support. Every detector cancels against g , because the two coordinates of each supported pair have equal weight. Every original at depth greater than one is disjoint from the support of g , while $g^T O_{1,i} = f^T M_i$. Consequently, $g^T F(M)$ is the indicator of the single physical occurrence of $O_{1,j}$ in W_1 . If $L = |W_2|$, that occurrence is at position

$$j(L + 1) = \frac{(n + 1)(L + 1)}{2} = \frac{n' + 1}{2},$$

because $n' + 1 = (n + 1)(L + 1)$. This proves (b) for the output and also identifies its central column.

It remains to verify (c). An original column $O_{\ell,i}$ occurs $(n + 1)^{\ell-1}$ times, once in each occurrence of W_ℓ , and each specified detector column $D_{\rho,i}$ occurs $(n + 1)^d$ times, once in each occurrence of Q . Since $n + 1$ is even, all these multiplicities are even except those of the depth-one originals, which occur once. Every source column is nonzero, so no detector equals an original: a detector has opposite nonzero entries on at least one coordinate pair, whereas an original has equal entries on every pair. Originals at different depths have disjoint nonzero supports. Collisions among detector types only add even multiplicities. Therefore the parity of the multiplicity of the type of $O_{1,i}$ equals the parity of the multiplicity of the type of M_i , and every other output type has even multiplicity. The map $M_i \mapsto O_{1,i}$ is injective on column types, so the unique

odd output type is the image of the unique odd source type. By the inductive hypothesis and the preceding identification of the central column, this is precisely the type of the central output column. This completes the simultaneous induction.

For every signing ε , (4.3) and the fact that f_k is a probability vector give

$$\|M_k \varepsilon\|_\infty \geq |f_k^\top M_k \varepsilon| = |\varepsilon_{j_k}| = 1. \quad (4.4)$$

Conversely, pair equal-column occurrences and give opposite signs to each pair, leaving one occurrence of the unique odd type unpaired. The final sum is that one unnormalized column and has ℓ_∞ -norm one. This proves (4.1) after normalization.

For (4.2), place each equal pair consecutively with opposite signs and put the unpaired occurrence last. Every unnormalized prefix is then either zero or one signed column, so its norm is at most one. Conversely, under any permutation and signing the first prefix has norm one. Division by $\sqrt{s_k}$ proves (4.2). $\square$

Use of AI: The author wrote a draft with an outline of the construction (split each coordinate into a mean and tracker, use nesting to freeze large means, try every possible row tuple and iterate) and fed this draft to ChatGPT and prompted it to work out the details, identify missing gaps and disprove Strong Komlós conjecture. The author fixed those missing gaps and iteratively verified and expanded the proofs with ChatGPT. The author is solely responsible for the correctness of the proofs.

References

- [1] W. Banaszczyk, *On series of signed vectors and their rearrangements*, Random Structures Algorithms 40 (2012), no. 3, 301–316.
- [2] N. Bansal, H. Jiang, R. Meka, S. Singla, and M. Sinha, *Prefix Discrepancy, Smoothed Analysis, and Combinatorial Vector Balancing*, in *13th Innovations in Theoretical Computer Science Conference*, LIPIcs 215, article 13, 2022. <https://doi.org/10.4230/LIPIcs.ITCS.2022.13>. Full version: arXiv:2111.07049, <https://arxiv.org/abs/2111.07049>.
- [3] S. Guo, E. X. Fang, and J. Lu, *Vector Balancing via Directional Total Variation*, arXiv:2609.11189, 2026. <https://arxiv.org/abs/2609.11189>.
- [4] S. R. Karingula and S. Lovett, *An Elementary Proof of the Komlós Conjecture*, arXiv:2609.20979, 2026. <https://arxiv.org/abs/2609.20979>.
- [5] J. Kulkarni, V. Reis, and T. Rothvoss, *Optimal Online Discrepancy Minimization*, in *Proceedings of the 56th Annual ACM Symposium on Theory of Computing (STOC 2024)*, 1832–1840, 2024. <https://doi.org/10.1145/3618260.3649720>. arXiv:2308.01406. <https://arxiv.org/abs/2308.01406>.
- [6] J. Spencer, *Balancing games*, J. Combin. Theory Ser. B 23 (1977), no. 1, 68–74.