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The determinantal complexity of the 4×4 permanent is 15

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Abstract

The best known lower bound on the determinantal complexity of the 4×4 permanent is 10. In this short note, I prove that this value is exactly 15: the 4×4 permanent is the determinant of a 15×15 matrix of affine linear forms, and cannot be that of any smaller matrix. To do so, I must establish both a lower and upper bound. To establish the upper bound, I use Grenet's construction of size $2^n - 1$. Hence that construction is optimal at $n = 4$ (and $n = 3$). This now constitutes the first known exact value of the determinantal complexity of the permanent at $n = 4$, improving over prior knowledge in which the exact value was known only for $n = 3$. Recall that the general quadratic bound only gives 8 for $n = 4$.

To establish the lower bound, I work from a normal form at a zero of the permanent. In the first row (and column) of the normal form, the linear forms vanish on a linear space inside the singular locus of the permanent. A hessian inequality bounds the size of the matrix in terms of that space. Then, a matrix of size 14 forces the space into any of nine coordinate types. The forcing is either up to symmetry, or else into a family that degenerates to one of them. I then rule out every case. All involved computations are exact.

In respect of the claim, I have made all certificates public.

This note was hand-written by the human author.

1 Introduction

The determinantal complexity $\mathrm{dc}(f)$ of a polynomial f is the smallest m for which $f = \det A$, where A is an $m \times m$ matrix whose entries are affine linear forms in the variables of f. Valiant showed that every polynomial computed by a small formula has small determinantal complexity, and that the permanent is complete for the class VNP [8]. His conjecture that $\mathrm{dc}(\mathrm{per}_n)$ grows faster than any polynomial in n is the determinant versus permanent form of the VP versus VNP question.

I consider the exact computation of small values of the permanent as revealing of it's behavior and of the pattern of growth of the permanent for larger, and asymptotically larger, values of n. It is my hope that a close study of the techniques that yield these values may one day cast a long enough shadow that at the least, a rough shape of the mathematics behind it may be deduced.

The known bounds are far apart. Mignon and Ressayre proved $\mathrm{dc}(\mathrm{per}_n) \geq n^2/2$ over $\mathbb{C}$ by comparing Hessians [6], and Cai, Chen and Li extended this to every characteristic other than 2 [3]. For $n = 3$, Alper, Bogart and Velasco proved that $\mathrm{dc}(\mathrm{per}_3) \geq 7$ [1]. On the upper bound side, Grenet gave a representation of size $2^n - 1$ [4]. These coincide at $n = 3$. Landsberg and Ressayre showed that $2^n - 1$ is optimal among representations that respect a large part of the symmetry group of the permanent [5], which leaves open whether a representation without that symmetry can be smaller. At $n = 4$ the quadratic bound gives 8 against Grenet's 15. An earlier stage of the present author's own analysis had recorded a best lower bound for $n = 4$ as 10.

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To the best of my knowledge, this is the first article that settles the exact case of $n = 4$ from both directions.

Theorem 1. $\mathrm{dc}(\mathrm{per}_4) = 15$ *over* $\mathbb{C}$.

The proof has four parts. Firstly, the singular locus of the permanent hypersurface has dimension 8. This forces every representation to have corank exactly one at every zero of per_4. It gives a normal form at the origin in which $\mathrm{per}_4 = bD^2c$, for a row b, a column c and a square matrix D of linear forms.

Secondly, the forms in b and c vanish together on a linear space K, which I call the endpoint kernel. The endpoint kernel lies in the singular locus. Let ρ be the rank of the Hessian of per_4 at a generic point of K. A short argument with a quadratic form then shows that the size of the matrix is at least $17 - \dim K + \lceil \rho/2 \rceil$.

Thirdly, at size 14 this inequality leaves nine coordinate kernels up to symmetry. The torus acting on rows and columns degenerates every other surviving kernel to one of them.

Finally, I exclude each surviving kernel at size 14. The hardest case is the kernel spanned by a single row. There the Hessian vanishes, and the inequality gives nothing beyond 13.

The argument depends on the characteristic. In characteristic 2 the permanent equals the determinant and $\mathrm{dc}(\mathrm{per}_4) = 4$. The step that fails there is the first one, because the singular locus of $\det_4$ has dimension 12.

The rest of this very short note is oragnized as follows. Section 2 derives the normal form and the endpoint inequality. Section 3 classifies the kernels that survive the inequality. Section 4 excludes them, and Section 5 lists the certificates.

2 Normal form and the endpoint inequality

This section reduces any representation of per_4 to a normal form at the origin and proves the inequality that the rest of the paper uses. We write $X = (x_{ij})$ for the 4×4 matrix of variables, so $\mathrm{per}_4 = \sum_\sigma \prod_i x_{i\sigma(i)}$ over all permutations σ of $\{1, 2, 3, 4\}$. The partial derivative $\partial \mathrm{per}_4 / \partial x_{ij}$ is the permanent of the 3×3 submatrix that omits row i and column j. Expanding per_4 along a row writes it as a combination of these subpermanents. Therefore the singular locus $\mathrm{Sing}(\mathrm{per}_4)$ of the hypersurface $\mathrm{per}_4 = 0$ is exactly the set where all sixteen 3×3 subpermanents vanish.

Lemma 2. $\mathrm{Sing}(\mathrm{per}_4)$ *has dimension* 8 *in* $\mathbb{C}^{16}$.

Proof. A Gröbner basis of the ideal of 3×3 subpermanents over $\mathbb{Q}$, computed with Singular [7], has Krull dimension 8. The script is in the archive (Section 5). □

A representation of size m is an $m \times m$ matrix $A(x)$ of affine linear forms in the sixteen variables with $\det A = \mathrm{per}_4$.

Lemma 3. *If* A *is a representation and* $\mathrm{per}_4(p) = 0$*, then* $\operatorname{rank} A(p) = m - 1$.

Proof. Let Δ be the set of $m \times m$ matrices of rank at most $m - 2$, and let $Z = A^{-1}(\Delta)$ be its preimage. Since Δ is irreducible of codimension 4, the codimension of every component of Z in $\mathbb{C}^{16}$ is at most 4 for nonempty Z. The gradient of det at a matrix of rank at most $m-2$ is zero, since its entries are $(m-1)$-minors. By the chain rule the gradient of per_4 vanishes on Z, so $Z \subseteq \mathrm{Sing}(\mathrm{per}_4)$. Lemma 2 gives codimension 8 for the singular locus, so Z is empty. Compare [9]. □

Note that since $\mathrm{per}_4(0) = 0$, the matrix $A(0)$ has rank $m-1$ by Lemma 3. Multiply A on both sides by constant invertible matrices, and then rescale one row. This enables us to assume that $A(0) = \mathrm{diag}(0, I_h)$ with $h = m-1$. This gives

$$A = \begin{pmatrix} \ell & -b \\ c & I_h - D \end{pmatrix},$$

where ℓ is a linear form, b is a row of h linear forms, c is a column of h linear forms and D is an $h \times h$ matrix of linear forms.

Lemma 4. *$\ell = 0$, $bc = 0$, $bDc = 0$ and $bD^2c = \mathrm{per}_4$.*

Proof. The Schur complement gives $\det A = \ell \det(I-D) + b\,\mathrm{adj}(I-D)\,c$. As formal power series, $\mathrm{adj}(I-D) = \det(I-D)(I-D)^{-1} = \big(\sum_i (-1)^i e_i\big)\big(\sum_j D^j\big)$, where e_i is the sum of the principal $i \times i$ minors of D and is homogeneous of degree i. Now compare homogeneous parts with per_4, which is homogeneous of degree 4. Degree 1 gives $\ell = 0$. Degrees 2, 3 and 4 then give $bc = 0$, $bDc - e_1 bc = 0$ and $bD^2c - e_1 bDc + e_2 bc = \mathrm{per}_4$. □

The endpoint map is $\Phi : \mathbb{C}^{16} \to W = \mathbb{C}^h \oplus \mathbb{C}^h$, $\Phi(y) = (b(y)^T, c(y))$. Its kernel K is the endpoint kernel of the representation, its image is U, and $a = \dim U = 16 - \dim K$. We give W the nondegenerate symmetric form $\langle(\beta,\gamma),(\beta',\gamma')\rangle = \beta^T\gamma' + \beta'^T\gamma$, and for $z \in \mathbb{C}^{16}$ we put $M_z(\beta,\gamma) = (D(z)^T\beta,\, D(z)\gamma)$. We write H_z for the Hessian of per_4 at z, $\rho(z)$ for its rank and ρ_K for the largest value of ρ on K.

Proposition 5. *$K \subseteq \mathrm{Sing}(\mathrm{per}_4)$, and $\rho(z) \le 2(h-a)$ for every $z \in K$. Therefore*

$$m \;\ge\; 17 - \dim K + \lceil \rho_K/2 \rceil.$$

Proof. Let $T(s_1,s_2,s_3,s_4) = b(s_1)D(s_2)D(s_3)c(s_4)$, so that $\mathrm{per}_4(x) = T(x,x,x,x)$. Define the trilinear form $R(s_1,s_2,s_3) = b(s_1)D(s_2)c(s_3)$ in the same way. For $z \in K$, every value of T or R with z in the first or last slot is zero. In particular $\mathrm{per}_4(z) = 0$. Now take the derivative of per_4 at z in a direction y. It is a sum of four values of T, each with z in three of the four slots. In each of them z occupies the first or the last slot, so each value is zero. Hence K lies in the singular locus.

Polarizing $bc = 0$ gives $\langle\Phi(y),\Phi(y')\rangle = b(y)c(y') + b(y')c(y) = 0$, so U is isotropic and $U \subseteq U^\perp$. Polarizing $bDc = 0$ at (y,z,y') with $z \in K$ leaves only the two orderings with z in the middle. Hence $b(y)D(z)c(y') + b(y')D(z)c(y) = 0$. The left side is $\langle M_z\Phi(y),\Phi(y')\rangle$, so $M_zU \subseteq U^\perp$.

Note that per_4 is homogeneous of degree 4, so $H_z(y,y') = \frac{1}{2}\,\partial_z\partial_z\partial_y\partial_{y'}\,\mathrm{per}_4$. Also, the fourth derivative is the sum of T over the 24 orderings of (y,y',z,z). Therefore,

$$H_z(y,y') = b(y)D(z)^2c(y') + b(y')D(z)^2c(y) = \langle M_z\Phi(y), M_z\Phi(y')\rangle.$$

Only the four orderings with y and y' at the ends survive, and they give the first equality. The form on W induces a nondegenerate form on $\overline{U} = U^\perp/U$, which has dimension $2h-2a$. The map $y \mapsto M_z\Phi(y) + U$ is linear into $\overline{U}$, and H_z is the pullback of that form along it. So $\rho(z) \le 2h-2a$, which is $h \ge 16 - \dim K + \rho(z)/2$. □

The inequality uses the Hessian in the spirit of [6], but at a point of the endpoint kernel rather than at a generic zero. It is weakest when $\rho_K = 0$, which happens when K is spanned by a row.

A representation of size $m < 14$ gives one of size 14 by adjoining an identity block, with the same endpoint kernel. So Theorem 1 follows from Grenet's construction and one remaining claim. Hence, the claim that no representation of size 14 exists is all that is left to prove. At $m = 14$ Proposition 5 requires $\dim K - \lceil \rho_K/2 \rceil \geq 3$. We call a linear subspace of $\mathrm{Sing}(\mathrm{per}_4)$ with this property admissible.

3 Admissible kernels

The endpoint inequality turns the question into one about linear spaces inside the singular locus. This section lists the admissible ones.

Two groups act. The group G of order 1152 generated by permutations of rows, permutations of columns and transposition preserves per_4. The torus $\mathbb{T} = (\mathbb{C}^*)^8$ acts by $x_{ij} \mapsto s_i t_j x_{ij}$ and multiplies per_4 by $\prod_i s_i \prod_j t_j$. Let g be an element of either group. Substitute g into a representation, and rescale one row. The result is another representation of the same size. Its endpoint kernel is the original kernel, transformed by g^{-1}. Both groups preserve $\mathrm{Sing}(\mathrm{per}_4)$ and Hessian ranks.

Let K_S be the span of the variables x_{ij} with $(i,j) \in S$, for a set S of cells. The restriction of $\partial \mathrm{per}_4 / \partial x_{ij}$ to K_S is a sum of distinct monomials. Each monomial corresponds to one permutation through (i,j) whose other three cells lie in S. Hence $K_S \subseteq \mathrm{Sing}(\mathrm{per}_4)$ exactly when every permutation has at most two of its cells in S.

Proposition 6. *Up to G there are 27 coordinate subspaces of $\mathrm{Sing}(\mathrm{per}_4)$, counting the zero space. Exactly nine are admissible. They are listed in Table 1.*

Proof. Enumerate all 2^{16} sets S, keep those that meet every permutation in at most two cells, and compute ρ as the rank of the Hessian with the variables outside S set to zero, over the field of rational functions. The script is in the archive. □

Kernels that are not coordinate subspaces come in continuous families, and the torus reduces them to the table.

Lemma 7. *The closure of the $\mathbb{T}$-orbit of an admissible kernel contains an admissible coordinate kernel.*

Proof. Let K have dimension d. The orbit closure of K in the Grassmannian of d-planes in $\mathbb{C}^{16}$ is projective and $\mathbb{T}$-stable, so it contains a fixed point by Borel's fixed point theorem [2]. The sixteen coordinates have pairwise distinct weights, so the fixed points are coordinate subspaces K_S. The d-planes inside $\mathrm{Sing}(\mathrm{per}_4)$ form a closed $\mathbb{T}$-stable set, so K_S lies in the singular locus. Hessian ranks are constant along the orbit and cannot rise in a limit, so $\rho_{K_S} \leq \rho_K$ and K_S is admissible. □

The table shows that each admissible coordinate kernel lies in one of three containers: a single row, two rows, or the complete cross spanned by row 1 and column 1.

Proposition 8. *Up to G, every admissible kernel lies in a single row, in two rows, or in the complete cross.*

Proof outline. The set of d-planes contained in some G-translate of a container is closed and $\mathbb{T}$-stable. Suppose that near each admissible coordinate kernel K_S, every d-plane inside $\mathrm{Sing}(\mathrm{per}_4)$

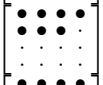
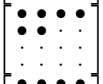
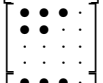
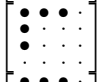
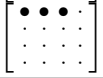
Kernel	Cells	$\dim K$	ρ_K	bound
two rows		8	8	13
full 2×3 block		6	4	13
one row		4	0	13
two rows less a cell		7	8	14
row and two cells		6	6	14
K_6		6	6	14
2×3 block less a cell		5	4	14
cross with arms of two		5	4	14
row less a cell		3	0	14

Table 1: The admissible coordinate kernels up to symmetry. The last column is the lower bound $17 - \dim K + \lceil \rho_K/2 \rceil$ of Proposition 5. The complete cross, a row together with a column, lies in the singular locus but forces size 15.

lies in a container. If K_S is in the orbit closure of K, then some $t \cdot K$ lies in that neighbourhood and therefore in a container, and so does $K = t^{-1} \cdot (t \cdot K)$. The local statement is proved in archive A by exact first order computations at the coordinate kernels where it is not immediate. The Jacobians of the incidence equations have full column rank 40, 48, 45 and 54 there, with certifying minors -64, 16, -26880 and -128. □

4 Excluding the admissible kernels

It remains to show that no representation of size 14 has an admissible endpoint kernel. We go container by container and leave the single row to the end, because there $\rho_K = 0$ and Proposition 5 carries no information beyond $m \geq 13$. The proofs in this section are condensed.

Theorem 9. *No representation of* per_4 *of size* 14 *has an admissible endpoint kernel.*

With Proposition 5, Theorem 9 gives $\mathrm{dc}(\mathrm{per}_4) \geq 15$, and Theorem 1 follows. Table 2 summarizes the cases.

4.1 Kernels inside two rows

Let the kernel lie in rows 1 and 2. Write z for the variables of those rows, and u, v for the variables of rows 3 and 4. Collect the terms of per_4 that have degree 2 in z, degree 1 in u and degree 1 in v. They form a polynomial $u^T B(z) v$, where $B(z)$ is a symmetric 4×4 matrix. Its diagonal is zero. For $j \neq k$, its (j, k) entry is the permanent of the 2×2 submatrix of rows 1 and

Container	Kernels	Mechanism
two rows	two rows, two rows less a cell	five channel factorization of B; Clifford reduction
	2×3 block and its hyperplanes	no linear syzygies among 2×2 subpermanents
	row and two cells	rank of the specialized Hessian block
cross	K_6	equality case of the inequality; irreducibility of per_3
	arms of two, other hyperplanes	rank certificate; odd factor obstruction
one row	row less a cell, its hyperplanes	nilpotence of D and irreducibility of F_z
	one row	invariant subspaces of a nilpotent pencil

Table 2: How each admissible kernel is excluded at size 14. Kernels inside a container include the noncoordinate ones.

2 that uses the two columns other than j and k. The determinant of B has degree 8. On the slice where row 1 is all ones, it equals $-4(e_1e_3 - 4e_4)$, where the e_i are the elementary symmetric functions of row 2.

Say the endpoints are separated if b depends only on u and c only on v.

Lemma 10. *Let K be spanned by rows 1 and 2, with separated endpoints, in a representation of size m. Then $B(z) = P(z)^T Q(z)$, where P and Q are $(m-9) \times 4$ matrices of linear forms.*

Proof. Write $b(u)$ for the row $\sum_j u_j \beta_j$ and $c(v)$ for the column $\sum_k v_k \gamma_k$. Since K is exactly the kernel, b is injective on u and c on v, so $U_\beta = b(\mathbb{C}^4)$ and $U_\gamma = c(\mathbb{C}^4)$ both have dimension 4. In bD^2c the slots of b and c must take u and v, because they vanish on z. Therefore the part of multidegree $(2,1,1)$ is $b(u)D(z)^2c(v)$. Let N_β be the space of rows β with $\beta\gamma = 0$ for every $\gamma \in U_\gamma$. Let N_γ be the space of columns γ with $\beta\gamma = 0$ for every $\beta \in U_\beta$. Isotropy of U gives $U_\beta \subseteq N_\beta$ and $U_\gamma \subseteq N_\gamma$, and $M_z U \subseteq U^\perp$ gives $b(u)D(z) \in N_\beta$ and $D(z)c(v) \in N_\gamma$. The product $\beta\gamma$ descends to a nondegenerate pairing between N_β/U_β and N_γ/U_γ. This is the form on $U^\perp/U$. Each quotient has dimension $h - 8 = m - 9$. Taking dual bases, $P(z)u$ is the class of $b(u)D(z)$ and $Q(z)v$ is the class of $D(z)c(v)$. □

At $m = 14$ this is a factorization of B through five channels.

Proposition 11. *B has no factorization P^TQ with P and Q of size 5×4 and linear in z.*

Proof outline. Archive A gives an exact classification into ten branches by the ranks of P and Q at special points. The locus where $B(1,z)$ has rank at most 2 is exactly ten lines, and each branch ends in a polynomial divisibility or an ideal membership that fails. □

When b or c involves both u and v, the endpoints are mixed, and Lemma 10 does not apply directly. The proof of Proposition 5 still factors the Hessian through the ten dimensional space $U^\perp/U$. We convert this into a linear factorization of B through at most five channels. The conversion uses an operator T_F on the exterior algebra of $\mathbb{C}^5$ for each isotropic frame F. These operators satisfy $\mathrm{tr}(T_F T_G) = \det(F^T J G)$ and $\operatorname{rank} T_F = 1$, where J is the Gram matrix of the form on $U^\perp/U$. Proposition 11 then applies. The other kernels inside two rows are excluded in archive A as Table 2 indicates.

4.2 Kernels inside the cross

For K_6 at $m = 14$ the inequality is an equality: $h - a = 3 = \rho/2$. Equality forces the classes $M_z\Phi(y) + U$ to fill $U^\perp/U$, which fixes the action of $D(z)$ on $U^\perp$ up to a Witt transformation.

Comparing the cubic part of $bD^2c = \mathrm{per}_4$ then forces the remaining part of D into a block form governed by one matrix G and one linear form ℓ. The coefficient of x_{11} in per_4 is the permanent per_3 of rows and columns $2, 3, 4$. Viewing per_3 as a trilinear function of its rows p, q, r, the resulting identity reads $\mathrm{per}_3(Gp, q, r) + \mathrm{per}_3(p, Gq, r) = 0$, and the linear operator $G \mapsto \mathrm{per}_3(Gp, q, r) + \mathrm{per}_3(p, Gq, r)$ is injective. So $G = 0$, and then per_3 is a linear form times a quadric. This contradicts the irreducibility of per_3.

The cross with arms of two is excluded by an exact rank certificate, and the noncoordinate hyperplanes of the cross by an obstruction on the factor degrees of a determinant of degree 8.

4.3 Kernels inside one row

Let K lie in row 1, with coordinates $z = (z_1, \ldots, z_4)$, and let Y denote rows 2 to 4. Then $\mathrm{per}_4 = F_z(Y) = \sum_j z_j \mathrm{per}_3(Y^{(j)})$, where $Y^{(j)}$ omits column j. For every $z \neq 0$ the cubic F_z is absolutely irreducible. Up to the torus and column permutations z is one of $(1,0,0,0)$, $(1,1,0,0)$, $(1,1,1,0)$ and $(1,1,1,1)$, and in each case F_z is irreducible over $\mathbb{Q}$ and has a smooth rational point, so it cannot split over $\mathbb{C}$. In particular F_z has no linear factor.

The kernels of dimension 3 in a row are excluded in archive A by a stabilizer computation, after which nilpotence of D on the kernel contradicts the irreducibility of F_z.

The full row is the last case. Here $h = 13$ and $a = 12$, so $U^\perp/U$ has dimension 2. Archive A reduces a representation of size 14 with this kernel to a nilpotent pencil $S(z) = \sum_j z_j S_j$ on a space $V = U \oplus \mathbb{C}e$ of dimension 13, together with symmetric matrices C_Y linear in Y. These are subject to three identities, which together express $F_z(Y)$ through S and C. Archive B shows that this system has no solution, without splitting into cases by the type of the pencil. The covectors on which the pencil can couple form ten lines and fifteen isolated points. On each of them, the rows of Y restricted to the kernel of the covector are eigenspaces of the whole pencil, so nilpotence forces the eigenvalues to vanish. What remains is $S(Y, 0) = \ell(Y)(p, 1)$ for a linear form ℓ, and one of the three identities then forces ℓ to divide F_z, which is impossible. When the coupling sits in a single row, the rows carrying b and c are invariant under the whole pencil, so its compressions to them are nilpotent. A trace computation removes the eigenvalue for each of the four covector types, and at an explicit endpoint where the remaining action vanishes one finds $F_z(Y) = 1$, -2 or -4, a contradiction.

5 Certificates

Every computational step is exact, and each comes with a checker that runs on its own.

Archive	SHA-256
A: `permanent4_exact_certificates.zip`	`f76999ef9f286b6d88795f5b93a1fe31` `73d0811d0c4720463a78ae085c51e21f`
B: `per4_type_independent_exclusion.zip`	`f6c4e772d3a9d3e392b17507510e6768` `9364bdd3302b411e360cc0221f8dd434`

The scripts `per4_singular_locus.sing` and `per4_checks.py` reproduce Lemma 2, Proposition 6, the formula for $\det B(1, z)$ and the irreducibility of F_z.

Acknowledgements

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